



JENN

Training and Consultancy

The path to enlightened education

VAK: WISKUNDE

GRAAD 12

2025 “LAST PUSH”

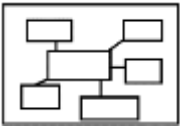





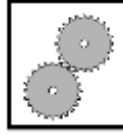

OPLOSSINGSHANDLEIDING

ONDERWERPE

1. ALLE VRAESTEL 1 ONDERWERPE

2. ALLE VRAESTEL 2 ONDERWERPE

IKON BESKRYWING

 <p>BREINKAART</p>	 <p>EKSAMEN RIGLYN</p>	 <p>INHOUD</p>	 <p>AKTIWITEITE</p>
 <p>BIBLIOGRAFIE</p>	 <p>TERMINOLOGIE</p>	 <p>UITGEWERKTE VOORBEELDE</p>	 <p>STAPPE</p>

**INHOUD****BLADSY**

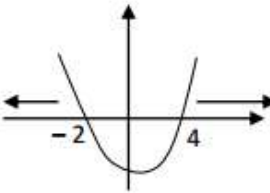
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BIBLIOGRAFIE	103

VRAESTEL 1

Algebra, Vergelykings en Ongelykhede

Mei/Junie 2024

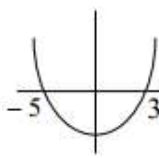
QUESTION 1/VRAAG 1

1.1.1	$3x^2 + 5x = 0$ $x(3x + 5) = 0$ $x = 0$ or $x = -\frac{5}{3}$	$\checkmark x = 0$ $\checkmark x = -\frac{5}{3}$ (2)
1.1.2	$4x^2 + 3x - 5 = 0$ $x = \frac{-(3) \pm \sqrt{(3)^2 - 4(4)(-5)}}{2(4)}$ $x = 0,80$ or $x = -1,55$	\checkmark correct substitution into correct formula \checkmark answer \checkmark answer (3)
1.1.3	$(x-1)^2 - 9 \geq 0$ $x^2 - 2x - 8 \geq 0$ $(x-4)(x+2) \geq 0$ $x = 4$ or $x = -2$ $x \leq -2$ or $x \geq 4$	 \checkmark standard form \checkmark critical values $\checkmark\checkmark x \leq -2$ or $x \geq 4$ (4)
1.1.4	$5^{2x} - 5^x = 0$ $5^x(5^x - 1) = 0$ $5^x \neq 0$ or $5^x = 1$ $x = 0$ OR/OF $5^{2x} = 5^x$ $2x = x$ $2x - x = 0$ $x = 0$	\checkmark common factor $\checkmark 5^x \neq 0$ $\checkmark 5^x = 1$ $\checkmark x = 0$ (4) OR/OF $\checkmark 5^{2x} = 5^x$ $\checkmark 2x = x$ $\checkmark 2x - x = 0$ $\checkmark x = 0$ (4)

1.1.5	$\frac{x}{\sqrt{20-x}} = 1$ $x = \sqrt{20-x}$ $x^2 = 20-x$ $x^2 + x - 20 = 0$ $(x+5)(x-4) = 0$ $x = 4 \text{ or } x = -5$	<ul style="list-style-type: none"> ✓ isolating the surd ✓ squaring both sides ✓ standard form ✓ answers ✓ selection
		(5)
1.2	$2x^2 - y^2 = 7 \quad \dots (1)$ $x + y = 9 \quad \dots (2)$ $y = 9 - x$ $2x^2 - (9-x)^2 = 7$ $2x^2 - 81 + 18x - x^2 = 7$ $x^2 + 18x - 88 = 0$ $(x+22)(x-4) = 0$ $x = -22 \text{ or } x = 4$ $y = 31 \text{ or } y = 5$ OR/OF $2x^2 - y^2 = 7 \quad \dots (1)$ $x + y = 9 \quad \dots (2)$ $x = 9 - y$ $2(9-y)^2 - y^2 = 7$ $2(81 - 18y + y^2) - y^2 - 7 = 0$ $162 - 36y + 2y^2 - y^2 - 7 = 0$ $y^2 - 36y + 155 = 0$ $(y-31)(y-5) = 0$ $y = 31 \text{ or } y = 5$ $x = -22 \text{ or } x = 4$	<ul style="list-style-type: none"> ✓ $y = 9 - x$ ✓ substitution ✓ standard form ✓ x-values ✓ y-values OR/OF <ul style="list-style-type: none"> ✓ $x = 9 - y$ ✓ substitution ✓ standard form ✓ y-values ✓ x-values
		(5)
1.3	$P \times T = (1-a)(1+a)(1+a^2)(1+a^4) \dots (1+a^{512})$ $P \times T = (1-a^2)(1+a^2)(1+a^4) \dots (1+a^{512})$ $P \times T = (1-a^4)(1+a^4) \dots (1+a^{512})$ $P \times T = (1-a^8) \dots (1+a^{512})$ $P \times T = (1-a^{512})(1+a^{512})$ $= 1 - a^{1024}$	<ul style="list-style-type: none"> ✓ $(1-a^4)$ ✓ $(1-a^{512})$ ✓ $1 - a^{1024}$
		(3)
		[26]

Mei/Junie 2023

QUESTION 1/VRAAG 1

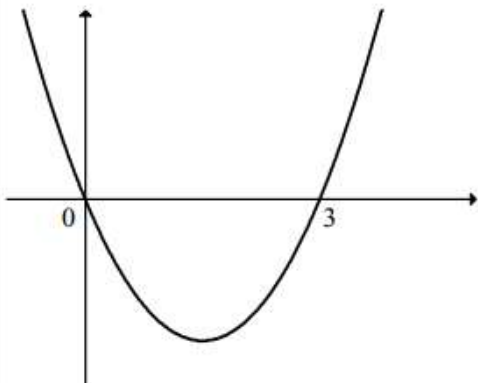
1.1.1	$x^2 - 7x + 12 = 0$ $(x-4)(x-3) = 0$ $x = 4$ or $x = 3$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Answer only: Full Marks </div>	✓ factors ✓ $x = 4$ ✓ $x = 3$	(3)
1.1.2	$3x^2 + 5x - 1 = 0$ $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-1)}}{2(3)} = \frac{-5 \pm \sqrt{37}}{6}$ $\therefore x = 0,18$ or $x = -1,85$		✓ standard form ✓ substitution into the correct formula ✓ $x = 0,18$ ✓ $x = -1,85$	(4)
1.1.3	$x^2 + 2x - 15 < 0$ $(x-3)(x+5) < 0$ $x = 3$ or $x = -5$ $-5 < x < 3$		✓ standard form ✓ critical values ✓ ✓ answer	(4)
1.1.4	$\sqrt{2(1-x)} = x-1$ $(\sqrt{2(1-x)})^2 = (x-1)^2$ $2-2x = x^2-2x+1$ $x^2-1=0$ $\therefore x=1$ and $x \neq -1$		✓ squaring both sides ✓ simplification ✓ standard form ✓ answer with selection	(4)

1.2	$3^{x+y} = 27$ $x^2 + y^2 = 17$ $3^{x+y} = 3^3$ $x + y = 3 \dots\dots(1)$ $y = 3 - x$ $x^2 + (3 - x)^2 = 17$ $2x^2 - 6x - 8 = 0$ $x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$ $x = 4$ or $x = -1$ $y = -1$ or $y = 4$ OR/OF $3^{x+y} = 27$ $x^2 + y^2 = 17$ $3^{x+y} = 3^3$ $x + y = 3 \dots\dots(1)$ $x = 3 - y$ $(3 - y)^2 + y^2 = 17$ $9 - 6y + y^2 + y^2 - 17 = 0$ $2y^2 - 6y - 8 = 0$ $y^2 - 3y - 4 = 0$ $(y - 4)(y + 1) = 0$ $y = -1$ or $y = 4$ $x = 4$ or $x = -1$	$\checkmark 3^{x+y} = 3^3$ $\checkmark x + y = 3$ \checkmark substitution \checkmark standard form $\checkmark x$ -values $\checkmark y$ -values (6) OR/OF $\checkmark 3^{x+y} = 3^3$ $\checkmark x + y = 3$ \checkmark substitution \checkmark standard form $\checkmark y$ -values $\checkmark x$ -values (6)
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1.3	$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}}$ $= \frac{1}{\sqrt{1}+\sqrt{2}} \times \frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}} + \dots$ $+ \frac{1}{\sqrt{99}+\sqrt{100}} \times \frac{\sqrt{99}-\sqrt{100}}{\sqrt{99}-\sqrt{100}}$ $= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + 2 \dots - \sqrt{99} + 10$ $= -1 + 10$ $= 9$	✓ rationalisation ✓ simplification ✓ answer (3)
		[24]

Mei/Junie 2022

QUESTION/VRAAG 1

1.1.1	$x^2 + 2x - 15 = 0$ $(x+5)(x-3) = 0$ $x = -5$ or $x = 3$	✓ factors ✓ $x = -5$ ✓ $x = 3$ (3)
1.1.2	$5x^2 - x - 9 = 0$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-9)}}{2(5)}$ $x = \frac{1 \pm \sqrt{181}}{10}$ $x = 1,45$ or $x = -1,25$	✓ substitution into the correct formula ✓ $x = 1,45$ ✓ $x = -1,25$ (3)
1.1.3	$x^2 \leq 3x$ $x^2 - 3x \leq 0$ $x(x-3) \leq 0$  $0 \leq x \leq 3$ OR $x \in [0;3]$	✓ standard form ✓ factors ✓✓ answer (4)

1.2.1	$a + \frac{64}{a} = 16$ $a^2 - 16a + 64 = 0$ $(a - 8)^2 = 0$ $a = 8$	✓ standard form ✓ factors ✓ answer (3)
1.2.2	$2^x + 2^{6-x} = 16$ $2^x + \frac{64}{2^x} = 16$ $2^x = 8 \text{ (from 1.2.1)}$ $2^x = 2^3$ $x = 3$	✓ exp law ✓ $2^x = 8$ ✓ answer (3)
1.3	$\sqrt{\frac{2^{1002}(1+2^4)}{17(2)^{998}}}$ $= \sqrt{\frac{2^4(17)}{17}}$ $= \sqrt{2^4}$ $= 2^2$ $= 4$	✓ common factor ✓ second factor ✓ simplification ✓ answer (4)

$$1.4 \quad 2x - y = 2 \quad \dots(1)$$

$$\frac{1}{x} - 3y = 1 \quad \dots(2)$$

$$y = 2x - 2$$

$$\checkmark y = 2x - 2$$

$$\frac{1}{x} - 3(2x - 2) = 1$$

✓ substitution

$$\frac{1}{x} - 6x + 6 - 1 = 0$$

✓ simplification

$$1 - 6x^2 + 6x - x = 0$$

$$-6x^2 + 5x + 1 = 0$$

✓ standard form

$$6x^2 - 5x - 1 = 0$$

$$(6x + 1)(x - 1) = 0$$

$$x = -\frac{1}{6} \quad \text{or} \quad x = 1$$

✓ x-values

$$y = 2\left(-\frac{1}{6}\right) - 2 \quad \text{or} \quad y = 2(1) - 2$$

$$y = -\frac{7}{3} \quad \text{or} \quad y = 0$$

✓ y-values

(6)

OR/OF

$$x = \frac{2+y}{2} \quad \dots(1)$$

$$\frac{1}{x} - 3y = 1 \quad \dots(2)$$

$$\frac{1}{\frac{2+y}{2}} - 3y = 1$$

$$\frac{2}{2+y} - 3y = 1$$

$$\frac{2-6y-3y^2}{2+y} = 1$$

$$2-6y-3y^2 = 2+y$$

$$-3y^2 - 7y = 0$$

$$-y(3y+7) = 0$$

$$y = 0 \quad \text{or} \quad y = -\frac{7}{3}$$

$$x = 1 \quad \text{or} \quad x = -\frac{1}{6}$$

OR/OF

$$\checkmark x = \frac{2+y}{2}$$

✓ substitution

✓ simplification

✓ standard form

✓ y-values

✓ x-values

(6)
[26]

Getalpatrone en Rye

Mei/Junie 2024

QUESTION 2/VRAAG 2

2.1.1	$r = \frac{1}{2}$ Yes, because $-1 < \frac{1}{2} < 1$	$\checkmark r = \frac{1}{2}$ \checkmark answer with reason (2)
2.1.2	$S_{\infty} = \frac{a}{1-r}$ $S_{\infty} = \frac{4}{1-\frac{1}{2}}$ $\therefore S_{\infty} = 8$	\checkmark substitution \checkmark answer (2)
2.2	$\sum_{p=k}^{10} 3^{p-1} = 3^{k-1} + 3^{k+1-1} + 3^{k+2-1} + \dots + 3^9$ $= 3^{k-1} + 3^k + 3^{k+1} + \dots + 3^9$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $29\,520 = \frac{3^{k-1}(3^{11-k} - 1)}{3 - 1}$ $3^{10} - 3^{k-1} = 59\,040$ $3^{k-1} = 9$ $k - 1 = 2$ $\therefore k = 3$	$\checkmark a = 3^{k-1}$ $\checkmark r = 3$ $\checkmark n = 11 - k$ \checkmark substitution \checkmark answer (5)
		[9]

QUESTION 3/VRAAG 3

<p>3.1.1</p> <p>3 ; 7 ; 12 ; 18 $\begin{array}{ccc} \vee & \vee & \vee \end{array}$</p> <p>First diff: 4 ; 5 ; 6 $\begin{array}{cc} \vee & \vee \end{array}$</p> <p>Second diff: 1 ; 1</p> <p>$2a=1$</p> <p>$a = \frac{1}{2}$</p> <p>$3a+b=4$</p> <p>$3\left(\frac{1}{2}\right)+b=4$</p> <p>$b = \frac{5}{2}$</p> <p>$a+b+c=3$</p> <p>$\frac{1}{2}+\frac{5}{2}+c=3$</p> <p>$c=0$</p> <p>$T_n = \frac{1}{2}n^2 + \frac{5}{2}n$</p>	<p>✓ $2a=1$</p> <p>✓ $3\left(\frac{1}{2}\right)+b=4$</p> <p>✓ $\frac{1}{2}+\frac{5}{2}+c=3$</p> <p>(3)</p>
<p>3.1.2</p> <p>$13\ 527 = \frac{1}{2}n^2 + \frac{5}{2}n$</p> <p>$n^2 + 5n - 27\ 054 = 0$</p> <p>$(n-162)(n+167)=0$</p> <p>$n=162$ or $n=-167$</p> <p>$T_{161} = 13\ 363$</p> <p>$\therefore T_{161} + 164 = 13\ 527$</p> <p>164 must be added.</p> <p>OR/OF</p> <p>$T_n = 3 + \text{sum of } 1^{\text{st}} \text{ differences}$</p> <p>$13\ 527 = 3 + 4 + 5 + \dots + n$</p> <p>$\frac{n-3+1}{2}[3+n] = 13\ 527$</p> <p>$n^2 + n - 27\ 060 = 0$</p> <p>$(n+165)(n-167)=0$</p> <p>$n = 164$</p>	<p>✓ $13\ 527 = \frac{1}{2}n^2 + \frac{5}{2}n$</p> <p>✓ standard form</p> <p>✓ answers for n</p> <p>✓ 164</p> <p>OR/OF</p> <p>✓ $13\ 527 = 3 + 4 + 5 + \dots + n$</p> <p>✓ $n^2 + n - 27\ 060 = 0$</p> <p>✓ answers for n</p> <p>✓ 164</p> <p>(4)</p>

3.2.1	$T_n = 8 + (n-1)(3)$ $T_n = 3n + 5$ $41 = 3n + 5$ $36 = 3n$ $n = 12$	✓ $T_n = 3n + 5$ ✓ $T_n = 41$ ✓ answer (3)
3.2.2a	$P_{41} = 12$	✓ answer (1)
3.2.2b	$P_8 = a + 7d = 1$ $P_{11} = a + 10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a + 7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$	✓ $a + 7d = 1$ ✓ $a + 10d = 2$ ✓ value of d ✓ value of a (4)
		[15]

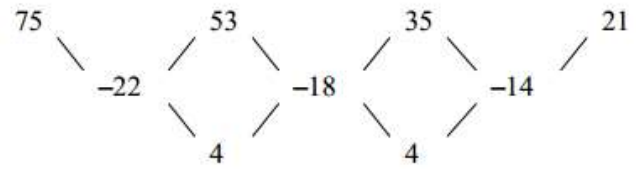
QUESTION 2/VRAAG 2

2.1.1	$\frac{1}{5} + \frac{1}{15} + \frac{1}{45} + \dots$ $r = \frac{\frac{1}{15}}{\frac{1}{5}} = \frac{1}{3}$ $-1 < \frac{1}{3} < 1$ $\therefore \text{the series is convergent.}$	$\checkmark r = \frac{1}{3}$ $\checkmark \text{ answer (any indicator of convergence) (2)}$
2.1.2	$S_{\infty} = \frac{a}{1-r}$ $= \frac{\frac{1}{5}}{1 - \frac{1}{3}}$ $= \frac{3}{10}$	$\checkmark \text{ substitution}$ $\checkmark \text{ answer (2)}$
2.2.1	$4x; \frac{1}{81}$	$\checkmark 4x \quad \checkmark \frac{1}{81} \quad (2)$
2.2.2	$T_n = x + (n-1)x$ $= x + xn - x$ $= xn$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">Answer only: Full Marks</div> $\checkmark \text{ substitution}$ $\checkmark \text{ answer (2)}$
2.2.3	$T_n = ar^{n-1}$ $T_{13} = \frac{1}{3} \left(\frac{1}{3} \right)^{13-1}$ $T_{13} = \left(\frac{1}{3} \right)^{13} \text{ or } \frac{1}{1\,594\,323} \text{ or } 6,27 \times 10^{-7} \text{ or } 3^{-13}$	$\checkmark n = 13$ $\checkmark r = \frac{1}{3}$ $\checkmark \text{ answer (3)}$
2.2.4	$\sum_{n=1}^{21} P_n = S_{11} + S_{10}$ $= \frac{11}{2} [2x + 10x] + \frac{\frac{1}{3} \left[1 - \left(\frac{1}{3} \right)^{10} \right]}{1 - \frac{1}{3}}$ $= 66x + 0,5$ $33,5 = 66x + 0,5$ $\therefore x = \frac{1}{2}$	$\checkmark S_{11} \checkmark + S_{10}$ $\checkmark \text{ arithmetic sum}$ $\checkmark \text{ geometric sum}$ $\checkmark 66x + 0,5 \quad (A)$ $\checkmark \text{ answer (6)}$
		[17]

QUESTION 3/VRAAG 3

<p>3.1</p>	$ \begin{array}{c} x \quad ; \quad x \quad ; \quad T_3 \quad ; \quad \dots \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 0 \quad \quad T_3 - x \\ \swarrow \quad \searrow \\ 10 \end{array} $ $ \begin{aligned} 2a &= 10 & 3a + b &= 0 \\ a &= 5 & b &= -15 \end{aligned} $ $ \begin{aligned} T_3 - x - 0 &= 10 \\ \therefore T_3 &= x + 10 \end{aligned} $ $ \begin{aligned} 2x + T_3 &= 28 \\ 2x + x + 10 &= 28 \\ 3x &= 18 \\ x &= 6 \end{aligned} $ $ \begin{aligned} a + b + c &= 6 \\ 5 - 15 + c &= 6 \\ c &= 16 \end{aligned} $ $\therefore T_n = 5n^2 - 15n + 16$ <p>OR/OF</p> $ \begin{aligned} 2a &= 10 \\ \therefore a &= 5 \end{aligned} $ $ \begin{aligned} T_1 &= a + b + c & T_2 &= 4a + 2b + c & T_3 &= 9a + 3b + c \\ &= 5 + b + c & &= 20 + 2b + c & &= 45 + 3b + c \end{aligned} $ $ \begin{aligned} 5 + b + c &= 20 + 2b + c \\ b &= -15 \end{aligned} $ $ \begin{aligned} T_1 &= -10 + c & T_2 &= -10 + c & T_3 &= c \end{aligned} $ $ \begin{aligned} T_1 + T_2 + T_3 &= -10 + c - 10 + c + c \\ 28 &= 3c - 20 \\ c &= 16 \end{aligned} $	$ \begin{aligned} \checkmark 2a &= 10 \\ \checkmark 3a + b &= 0 \end{aligned} $ $\checkmark T_3 = x + 10$ $\checkmark 2x + T_3 = 28$ $\checkmark x = 6$ $\checkmark 5 - 15 + c = 6$ <p style="text-align: right;">(6)</p> <p>OR/OF</p> $\checkmark 2a = 10$ $\checkmark 5 + b + c = 20 + 2b + c$ $ \begin{aligned} \checkmark T_1 &= -10 + c \\ \checkmark T_2 &= -10 + c \end{aligned} $ $ \begin{aligned} \checkmark 28 &= 3c - 20 \\ \checkmark c &= 16 \end{aligned} $ <p style="text-align: right;">(6)</p>
<p>3.2</p>	$ \begin{aligned} T_n &= 5n^2 - 15n + 16 \\ 216 &= 5n^2 - 15n + 16 \\ 5n^2 - 15n - 200 &= 0 \\ n^2 - 3n - 40 &= 0 \\ (n - 8)(n + 5) &= 0 \\ n = 8 \quad \text{or} \quad n = -5 \\ \therefore T_8 &= 216 \end{aligned} $	$\checkmark \text{equating}$ $\checkmark \text{standard form}$ $\checkmark n = 8$ <p style="text-align: right;">(3)</p>
	[9]	

QUESTION/VRAAG 2

2.1.1	$a + 6d = 35$ $-1 + 6d = 35$ $6d = 36$ $d = 6$ OR/OF $\frac{35 - (-1)}{7 - 1} = 6$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;">ANSWER ONLY: FULL MARKS</div>	✓ substitution ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
2.1.2	$T_n = a + (n - 1)d$ $473 = -1 + (n - 1)(6)$ $79 = n - 1$ $\therefore n = 80$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;">ANSWER ONLY: FULL MARKS</div>	✓ substitution into the correct formula ✓ equating to 473 ✓ answer (3)
2.1.3	$S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_{40} = \frac{40}{2}[2(-1) + (40 - 1)(6)]$ $\therefore S_{40} = 4640$ OR/OF $T_{40} = 6(40) - 7$ $= 233$ $S_n = \frac{n}{2}(a + l)$ $= \frac{40}{2}(-1 + 233)$ $= 4640$		✓ substitution ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
2.2.1	 $T_5 = 11$		✓ answer (A) (1)
2.2.2	$T_n = an^2 + bn + c$ $2a = 4$ $a = 2$ $3a + b = -22$ $6 + b = -22$ $b = -28$ $a + b + c = 75$ $2 - 28 + c = 75$ $c = 101$ $\therefore T_n = 2n^2 - 28n + 101$		✓ $T_n = an^2 + bn + c$ ✓ $a = 2$ ✓ $b = -28$ ✓ $c = 101$ (4)

2.2.3

Minimum value of T_n

$$n = -\frac{b}{2a} = -\frac{(-28)}{2(2)}$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

OR/OF

$$T'_n = 4n - 28$$

$$4n - 28 = 0$$

$$4n = 28$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

$$\checkmark n = 7$$

$$\checkmark \text{ min value} = 3$$

$$\checkmark -\frac{1}{5} \text{ value of term of old pattern}$$

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$\checkmark n = 7$$

$$\checkmark \text{ min value} = 3$$

$$\checkmark -\frac{1}{5} \text{ value of term of old pattern}$$

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$T'_n = 4n - 28$$

$$4n - 28 = 0$$

$$4n = 28$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

OR/OF

$$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$$

$$n = -\frac{b}{2a} = \frac{-\frac{28}{5}}{2\left(-\frac{2}{5}\right)}$$
$$= 7$$

$$T_7 = -\frac{3}{5}$$

OR/OF

OR/OF

$$\checkmark n = 7$$

$$\checkmark \text{ min value} = 3$$

$$\checkmark -\frac{1}{5} \text{ value of term of old pattern}$$

$$\checkmark \text{ max value} = -\frac{3}{5}$$

(4)

OR/OF

$$\checkmark \checkmark T_n \div (-5)$$

$$\checkmark n = 7$$

$$\checkmark \text{ max value} = -\frac{3}{5} \quad (4)$$

OR/OF

$$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$$

$$T'_n = -\frac{4}{5}n + \frac{28}{5}$$

$$\checkmark\checkmark T_n \div (-5)$$

$$-\frac{4}{5}n + \frac{28}{5} = 0$$

$$-4n = -28$$

$$n = 7$$

$$\text{Minimum value of } T_n = 2(7)^2 - 28(7) + 101 = 3$$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

$$\checkmark n = 7$$

$$\text{Maximum value of new pattern} = -\frac{3}{5}$$

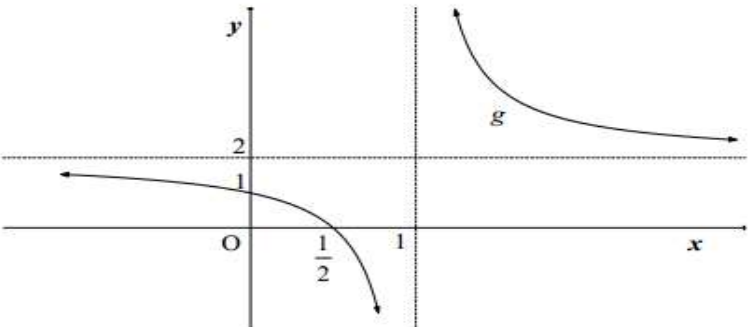
$$\checkmark \text{ max value} = -\frac{3}{5}$$

(4)

[16]

Funksies en Grafieke

QUESTION 4/VRAAG 4

4.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.2		✓ x-intercept ✓ y-intercept ✓ asymptotes ✓ shape (4)
4.3	$x < \frac{1}{2}$ or $x > 1$	✓ $x < \frac{1}{2}$ ✓ $x > 1$ (2)
4.4	$y = -(x - 1) + 2$ $y = -x + 3$ OR/OF $y - 2 = -(x - 1)$ $y = -x + 3$ OR/OF $y = -x + c$ $2 = -(1) + c$ $c = 3$ $\therefore y = -x + 3$	✓ substitution of (1 ; 2) ✓ answer (2) OR/OF ✓ substitution of (1 ; 2) ✓ answer (2) OR/OF ✓ substitution of (1 ; 2) ✓ answer (2)
		[10]

QUESTIONS/VRAAG 5

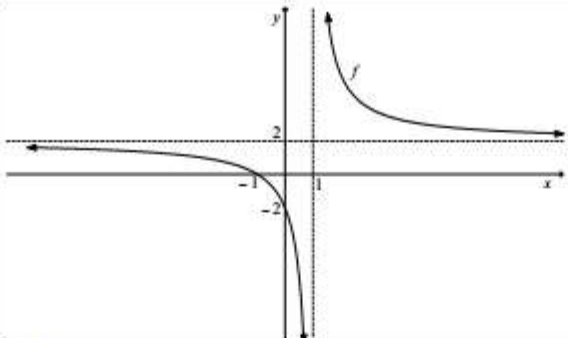
5.1	$P'(2; 4)$	$\checkmark x = 2$ $\checkmark y = 4$ (2)
5.2	$f(x) = \log_a x$ $2 = \log_a 4$ $a^2 = 4$ $a = 2$	\checkmark substitute (4 ; 2) $\checkmark a^2 = 4$ (2)
5.3	$y = 2^x$	$\checkmark y = 2^x$ (1)
5.4	$1 = \log_2 x$ $\therefore x = 2$ $T(2; 1)$ RT = 2 units P'T = 3 units Area of $\Delta RTP' = \frac{1}{2} \cdot RT \cdot TP'$ $= \frac{1}{2} \times 2 \times 3 = 3 \text{ units}^2$	$\checkmark x = 2$ $\checkmark RT = 2 \text{ units}$ $\checkmark P'T = 3 \text{ units}$ \checkmark answer (4)
		[9]

QUESTION 6/VRAAG 6

6.1	$y \geq -4$ or $y \in [-4; \infty)$	✓ answer (1)
6.2	$x^2 - 2x - 3 = 0$ $(x - 3)(x + 1) = 0$ $x = 3$ or $x = -1$ $\therefore E(3; 0)$ and $D(-1; 0)$	✓ = 0 ✓ both x-values ✓ correct identification of coordinates (3)
6.3	$P(0; -3)$ $\therefore m_{PE} = 1$ $\therefore g(x) = x - 3$	✓ $m_{PE} = 1$ ✓ $g(x) = x - 3$ (2)
6.4	$f(x) > g(x)$ $x < 0$ or $x > 3$	✓ $x < 0$ ✓ $x > 3$ (2)
6.5	Distance $= -x^2 + 2x + 3 - x + 3 = -x^2 + x + 6$ $D' = -2x + 1 = 0$ or/of $x = -\frac{b}{2a}$ $\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}$ $D\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $= \frac{25}{4} = 6,25$	✓ $D = -x^2 + x + 6$ ✓ method ✓ $x = \frac{1}{2}$ ✓ substitution ✓ answer (5)
6.6	$f'(x) = m$ $1 = 2x - 2$ $x = \frac{3}{2}$ Point on f : $\left(\frac{3}{2}; -\frac{15}{4}\right)$ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ $\therefore n = 2\frac{1}{4} = \frac{9}{4} = 2,25$	✓ $1 = 2x - 2$ ✓ $x = \frac{3}{2}$ ✓ $-\frac{15}{4}$ ✓ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ ✓ answer (5)
	OR/OF $f(x) = k(x)$ $x^2 - 2x - 3 = x - 3 - n$ $x^2 - 3x + n = 0$ $\Delta = b^2 - 4ac$ $= (-3)^2 - 4(1)(n)$ To touch: $\Delta = 0$ $0 = 9 - 4n$ $4n = 9$ $n = \frac{9}{4}$	OR/OF ✓ equating ✓ standard form ✓ substitution into Δ ✓ $\Delta = 0$ ✓ answer (5)
		[18]

QUESTION 4/VRAAG 4

4.1.1	decreasing	✓ decreasing (1)
4.1.2	$y = \left(\frac{1}{3}\right)^x$ $x = \left(\frac{1}{3}\right)^y$ $\therefore y = \log_{\frac{1}{3}} x$ OR/OF $y = 3^{-x}$ $x = 3^{-y}$ $\therefore y = -\log_3 x$	✓ swop x and y ✓ answer (2) OR/OF ✓ swop x and y ✓ answer (2)
4.1.3	$x > 0; x \in R$	✓ answer (1)
4.1.4	$y = -5$	✓ answer (1)
4.2.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.2.2	$\frac{4}{x-1} + 2 = 0$ $4 = -2x + 2$ $2x = -2$ $x = -1$	✓ let $y = 0$ ✓ $x = -1$ (2)

4.2.3		<ul style="list-style-type: none"> ✓ asymptotes ✓ x-intercept ✓ y-intercept ✓ shape <p>(4)</p>
4.2.4	$\frac{4}{x-1} \geq -2$ $\frac{4}{x-1} + 2 \geq 0$ $x \leq -1 \quad \text{or} \quad x > 1$	<ul style="list-style-type: none"> ✓ $x \leq -1$ ✓ $x > 1$ <p>(2)</p>
4.2.5	$y = -x + c$ $2 = -3 + c$ $c = 5$ $y = -x + 5$ <p>OR/OF</p> $y = -x + c$ $2 = -1 + c$ $c = 3$ $y = -x + 3$ $y = -(x-2) + 3$ $y = -x + 5$ <p>OR/OF</p> $y = -(x+p) + q$ $y = -((x-2) + (-1)) + 2$ $y = -x + 5$	<ul style="list-style-type: none"> ✓ intersection of axes at (3 ; 2) ✓ subst (3 ; 2) and $m = -1$ ✓ $y = -x + 5$ <p>(3)</p> <p>OR/OF</p> <ul style="list-style-type: none"> ✓✓ $-(x-2) + 3$ ✓ $y = -x + 5$ <p>(3)</p> <p>OR/OF</p> <ul style="list-style-type: none"> ✓✓ $y = -((x-2) + (-1)) + 2$ ✓ $y = -x + 5$ <p>(3)</p>
		[18]

QUESTION 5/VRAAG 5

5.1	T.P(-3;4)	✓ -3 ✓ 4 (2)
5.2	$y \leq 4$ or $y \in (-\infty; 4]$	✓ answer (1)
5.3	$f(x) = g(x)$ $-(x+3)^2 + 4 = x+5$ $-x^2 - 6x - 9 + 4 = x+5$ $-x^2 - 7x - 10 = 0$ $x^2 + 7x + 10 = 0$ $(x+5)(x+2) = 0$ $x = -5$ or $x = -2$	✓ equating ✓ $-x^2 - 6x - 9$ ✓ standard form ✓ factors (4)
5.4	The graph must shift more than 2 and less than 5 units to the right $\therefore -5 < c < -2$	✓✓ answer (2)
5.5	$D(x) = f(x) - g(x) = -x^2 - 7x - 10$ Max: $-2x - 7 = 0$ OR/OF $x = \frac{-(-7)}{2(-1)}$ $x = -\frac{7}{2}$ $D\left(-\frac{7}{2}\right) = -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10 = 2,25$ $\therefore k = 2,25$ $\therefore h(x) = x + 7,25$	✓ distance ✓ $-2x - 7 = 0$ ✓ $x = -\frac{7}{2}$ ✓ $k = 2,25$ ✓ $h(x) = x + 7,25$ (5)
		[14]

QUESTION 4

4.1	$10 = a\left(\frac{1}{3}\right)^{-2} + 7$ $3 = 9a$ $\therefore a = \frac{1}{3}$	✓ subs $(-2 ; 10)$ ✓ simplification ✓ answer (3)
4.2	$y = g(0)$ $y = \frac{1}{3} \times \left(\frac{1}{3}\right)^0 + 7$ $y = \frac{22}{3} = 7,33$ $\therefore \left(0 ; \frac{22}{3}\right)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">ANSWER ONLY: FULL MARKS</div>	✓ substitution of $x = 0$ ✓ answer (2)
4.3.1	Translation by 1 unit to the right and 7 units downwards	✓ 1 unit right ✓ 7 units downwards (2)
4.3.2	$h(x) = \left(\frac{1}{3}\right)^x$ $h^{-1}: x = \left(\frac{1}{3}\right)^y$ $y = \log_{\frac{1}{3}}(x) \quad \text{OR/OR} \quad y = -\log_3(x)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">ANSWER ONLY: FULL MARKS</div>	✓ swap x and y ✓ answer (2)
		[9]

QUESTION 5

5.1	$g(x) = \frac{a}{x+2} + q$ Subs (1 ; 0): $0 = \frac{a}{1+2} + q$ $0 = a + 3q$ Subs $\left(0 ; -\frac{1}{2}\right)$ $-\frac{1}{2} = \frac{a}{0+2} + q$ $-1 = a + 2q$ Solving simultaneously: $q = 1$ $a = -3$ $\therefore g(x) = \frac{-3}{x+2} + 1$	$\checkmark g(x) = \frac{a}{x+2} + q$ $\checkmark 0 = a + 3q$ $\checkmark -1 = a + 2q$ \checkmark solving simultaneously $\checkmark q = 1$ $\checkmark a = -3$ (6)
5.2	$y \in \mathbb{R}; y \neq 1$ OR/OF $(-\infty; 1)$ or $(1; \infty)$ OR/OF $y < 1$ or $y > 1$	\checkmark answer (1)
5.3	$y - 1 = 1(x + 2)$ OR/OF $1 = 1(-2) + c$ $y = x + 3$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">ANSWER ONLY: FULL MARKS</div> $c = 3$ $y = x + 3$	$\checkmark m = 1$ \checkmark subs point $(-2; 1)$ \checkmark answer (3)
5.4	$K'(-3; 4)$	\checkmark x-value \checkmark y-value (2)
		[12]

QUESTION 6

6.1	$f(x) = -x^2 - 6x + 7$ $f'(x) = -2x - 6$ $-2x - 6 = 0$ $x = -3$ $E(-3 ; 16)$	<p>OR/OF</p> $x = -\frac{(-6)}{2(-1)}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">ANSWER ONLY: FULL MARKS</div>	✓ method ✓ x-value ✓ y-value (3)
6.2	$k = f(-5)$ $k = -(-5)^2 - 6(-5) + 7$ $\therefore k = 12$		✓ answer (A) (1)
6.3	$C(0 ; 7)$ $D(-5 ; 12)$ $m_{CD} = \frac{12 - 7}{-5 - 0}$ $m_{CD} = -1$ Equation of CD: $y = -x + 7$		✓ coordinates of C ✓ substitution ✓ m ✓ answer (4)
6.4	$-2x - 6 = -1$ $-2x = 5$ $x = -\frac{5}{2}$ $y = f\left(-\frac{5}{2}\right) = -\left(-\frac{5}{2}\right)^2 - 6\left(-\frac{5}{2}\right) + 7 = \frac{63}{4} = 15,75$ $\therefore P\left(-\frac{5}{2}; \frac{63}{4}\right)$		✓ $f'(x) = -2x - 6$ ✓ equating to -1 ✓ x-value ✓ y-value (A) (4)
6.5	Point by symmetry: $(-1 ; 12)$ $-5 < x < -1$ OR/OF $-x^2 - 6x + 7 > 12$ $-x^2 - 6x - 5 > 0$ $x^2 + 6x + 5 < 0$ $(x+1)(x+5) < 0$ $-5 < x < -1$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">ANSWER ONLY: FULL MARKS</div>	✓ -1 ✓ answer (2) ✓ -1 ✓ answer (2)
			[14]

Finansies, Groei en Verval

Mei/Junie 2024

QUESTION 7/VRAAG 7

7.1	$A = P(1-i)^n$ $8\,337,75 = 13\,000(1-i)^6$ $i = 7,14\%$	<p>✓ substitution in correct formula</p> <p>✓✓ answer</p> <p>(3)</p>
7.2	$F = \frac{x[(1+i)^n - 1]}{i}$ $80\,000 = \frac{x\left[\left(1 + \frac{8,6}{1200}\right)^{36} - 1\right]}{\frac{8,6}{1200}}$ $x = R1\,955,78$ <p>Thandi's total = $1955,78 \times 36 = R\,70\,408,08$</p> <p>Eric's total = $1402,31 \times 48 = R\,67\,310,88$</p> <p>Difference = $70\,409,08 - 67\,310,88$ $= R3\,097,20$</p>	<p>✓ i</p> <p>✓ substitution into correct formula</p> <p>✓ answer</p> <p>✓ answer</p> <p>(4)</p>
7.3	$A = P(1+i)^n$ $A = 225\,000\left(1 + \frac{0,09}{12}\right)^3$ $A = R\,230\,100,5637\dots$ $225\,000\left(1 + \frac{0,09}{12}\right)^3 = \frac{5\,500\left[1 - \left(1 + \frac{0,09}{12}\right)^{-n}\right]}{\frac{0,09}{12}}$ $0,3137734959\dots = 1 - \left(1 + \frac{0,09}{12}\right)^{-n}$ $\left(1 + \frac{0,09}{12}\right)^{-n} = 0,6862265041\dots$ $-n = \log_{\left(1 + \frac{0,09}{12}\right)} 0,6862265041\dots$ $n = 50,394375\dots$ $n = 51$	<p>✓ substitution in correct formula</p> <p>✓ answer</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ use of logs</p> <p>✓ answer</p> <p>(6)</p>
		[13]

Mei/Junie 2023

QUESTION 6/VRAAG 6

6.1.1	$A = P(1+i)^n$ $A = 150\,000(1+0,065)^5$ $A = R205\,513$	✓ substitution into the correct formula ✓ answer (2)
6.1.2	$A = P(1-in)$ $A = 150\,000(1-0,09 \times 5)$ $A = 150\,000 - 67\,000$ $A = R82\,500$	✓ substitution into the correct formula ✓ answer (2)
6.1.3	$SF = A - T = 205\,513 - 82\,500$ $= R123\,013$ $F = \frac{x[(1+i)^n - 1]}{i}$ $x = \frac{F \times i}{(1+i)^n - 1}$ $x = \frac{123\,013 \times \frac{0,0785}{12}}{\left[\left(1 + \frac{0,0785}{12}\right)^{59} - 1\right] \left(1 + \frac{0,0785}{12}\right)}$ $= R1\,704,01$	✓ answer ✓ $i = \frac{0,0785}{12}$ ✓ 59 and $\left(1 + \frac{0,0785}{12}\right)$ (A) ✓ answer (A) (4)
6.2	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $200\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,0525}{4}\right)^{-4n}\right]}{\frac{0,0525}{4}}$ $\frac{7}{16} = 1 - \left(1 + \frac{0,0525}{4}\right)^{-4n}$ $\frac{9}{16} = \left(\frac{1621}{1600}\right)^{-4n}$ $-4n = \frac{\log \frac{9}{16}}{\log \left(\frac{1621}{1600}\right)}$ $-4n = -44,1243 \dots$ $n = 11,03 \text{ years}$	✓ substitution into correct formula ✓ simplification ✓ use of logs ✓ $-4n = -44,1243 \dots$ ✓ $n = 11,03 \text{ years}$ (5)
		[13]

QUESTION 7

7.1	$A = P(1+i)^n$ $2 = 1\left(1 + \frac{0,085}{4}\right)^{4n}$ $4n = \log_{\left(1 + \frac{0,085}{4}\right)} 2$ $n = 8,24 \text{ years}$	$\left. \begin{array}{l} \checkmark 2 \\ \checkmark \frac{0,085}{4} \end{array} \right\}$ In correct formula \checkmark use of logs \checkmark answer in years (4)
7.2.1	$A = P(1-i)^n$ $180\,000 = 500\,000(1-i)^5$ $\frac{9}{25} = (1-i)^5$ $\sqrt[5]{\frac{9}{25}} = 1-i$ $i = 0,1848068\dots$ $r = 18,48\%$	\checkmark subs into correct formula \checkmark simplification $\checkmark i = 0,1848\dots$ \checkmark answer (4)
7.2.2	$A = P(1+i)^n$ $A = 500\,000(1+0,063)^5$ $A = R678\,635,11$	\checkmark subs into correct formula \checkmark answer (2)
7.2.3	Sinking Fund = $678\,635,11 - 180\,000$ = R 498 635,11 $498\,635,11 = \frac{x \left[\left(1 + \frac{0,1025}{12}\right)^{58} - 1 \right] \left(1 + \frac{0,1025}{12}\right)^3}{\frac{0,1025}{12}}$ $x = R6\,510,36$	\checkmark value of sinking fund $\checkmark \frac{0,1025}{12}$ $\checkmark n = 58 \text{ (A)}$ $\checkmark \left(1 + \frac{0,1025}{12}\right)^3$ \checkmark answer (A) (5)
		[15]

Differensiale Rekenen

Mei/Junie 2024

QUESTION8/VRAAG 8

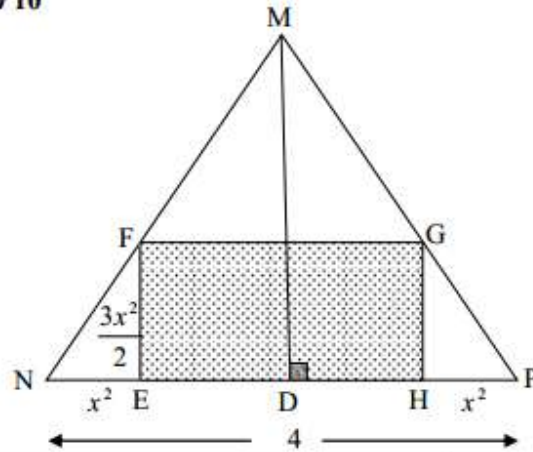
8.1	$f(x) = \frac{1}{x}$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{x - (x+h)}{x(x+h)} \times \frac{1}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$ $f'(x) = -\frac{1}{x^2}$ <p>OR/OF</p> $f(x) = \frac{1}{x}$ $f(x+h) = \frac{1}{x+h}$ $f(x+h) - f(x) = -\frac{h}{x(x+h)}$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$ $f'(x) = -\frac{1}{x^2}$	$\checkmark f(x+h) = \frac{1}{x+h}$ $\checkmark \frac{x - (x+h)}{x(x+h)} \times \frac{1}{h}$ $\checkmark \frac{-h}{x(x+h)} \times \frac{1}{h}$ $\checkmark \frac{-1}{x(x+h)}$ <p>\checkmark answer</p> <p>(5)</p> <p>OR/OF</p> $\checkmark f(x+h) = \frac{1}{x+h}$ $\checkmark f(x+h) - f(x) = -\frac{h}{x(x+h)}$ $\checkmark \frac{-h}{x(x+h)} \times \frac{1}{h}$ $\checkmark \frac{-1}{x(x+h)}$ <p>\checkmark answer</p> <p>(5)</p>
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8.2.1	$\frac{d}{dx}(\sqrt{4x^6} + \sqrt{2} \cdot x^2)$ $= \frac{d}{dx}(2x^3 + \sqrt{2} \cdot x^2)$ $= 6x^2 + 2\sqrt{2}x$	$\checkmark 2x^3$ $\checkmark 6x^2$ $\checkmark 2\sqrt{2}x$	(3)
8.2.2	$g(x) = \frac{3x^4 - 4x^2 + 6}{x^2}$ $g(x) = 3x^2 - 4 + 6x^{-2}$ $g'(x) = 6x - 12x^{-3}$	$\checkmark 3x^2 - 4 + 6x^{-2}$ $\checkmark 6x$ $\checkmark -12x^{-3}$	(3)
8.3	$f(x) = 3x^2 + bx + c$ $f'(x) = 6x + b$ $f'(1) = 6 + b = 9$ $\therefore b = 3$ $f(1) = 3 + 3 + c = 0$ $c = -6$ $\therefore f(x) = 3x^2 + 3x - 6$	$\checkmark f'(1) = 6 + b = 9$ $\checkmark b = 3$ $\checkmark f(1) = 3 + 3 + c = 0$ $\checkmark c = -6$	(4)
			[15]

QUESTION9/VRAAG 9

9.1	$f(x) = ax^3 + bx^2 + cx - 5$ $-5 = a(0+1)^2(0-5)$ $-5 = -5a$ $a = 1$ $f(x) = (x+1)(x+1)(x-5)$ $f(x) = (x^2 + 2x + 1)(x-5)$ $f(x) = x^3 - 3x^2 - 9x - 5$ $\therefore b = -3$ and $c = -9$	✓ substitution of x-intercepts ✓ simplification ✓ simplification (3)
9.2	$f(x) = x^3 - 3x^2 - 9x - 5$ $f'(x) = 3x^2 - 6x - 9$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ Minimum value at $x = 3$	✓ $f'(x) = 3x^2 - 6x - 9$ ✓ $f'(x) = 0$ ✓ factors ✓ $x = 3$ (4)
9.3	$f''(x).f(x) > 0$ Point of inflection: $x = 1$ $x < 1$; $x \neq -1$ or $x > 5$	✓ $x = 1$ ✓ $x < 1$; $x \neq -1$ ✓ $x > 5$ (3)
9.4	$-32 < -t < -5$ $5 < t < 32$ <p style="text-align: center;">OR/OF</p> Shift up more than 5 units and less than 32 units $\therefore 5 < t < 32$	✓ -32 ✓ $-32 < -t < -5$ ✓ $5 < t < 32$ (3) <p style="text-align: center;">OR/OF</p> ✓ more than 5 units ✓ less than 32 units ✓ $5 < t < 32$ (3)
		[13]

QUESTION 10/VRAAG 10

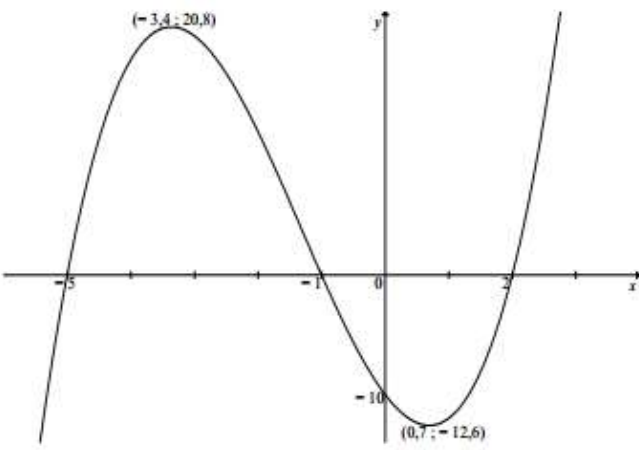


10.1	$\frac{NE}{EF} = \frac{2}{3} = \frac{x^2}{b}$ $3x^2 = 2b$ $\therefore b = \frac{3x^2}{2}$ $EH = 4 - 2x^2$ $\text{Area EFGH} = (4 - 2x^2) \left(\frac{3x^2}{2} \right)$ $A(x) = 6x^2 - 3x^4$ <p>OR/OF</p> <p>In $\triangle DMP$: $\tan P = \frac{3}{2}$</p> <p>In $\triangle HGP$: $\tan P = \frac{GH}{x^2}$</p> $\frac{GH}{x^2} = \frac{3}{2}$ $\therefore b = \frac{3x^2}{2}$ $EH = 4 - 2x^2$ $\text{Area EFGH} = (4 - 2x^2) \left(\frac{3x^2}{2} \right)$ $A(x) = 6x^2 - 3x^4$	$\checkmark \frac{NE}{EF} = \frac{2}{3} = \frac{x^2}{b}$ $\checkmark \therefore b = \frac{3x^2}{2}$ $\checkmark EH = 4 - 2x^2$ $\checkmark (4 - 2x^2) \left(\frac{3x^2}{2} \right)$ <p style="text-align: right;">(4)</p> <p>OR/OF</p> $\checkmark \frac{GH}{x^2} = \frac{3}{2}$ $\checkmark \therefore b = \frac{3x^2}{2}$ $\checkmark EH = 4 - 2x^2$ $\checkmark (4 - 2x^2) \left(\frac{3x^2}{2} \right)$ <p style="text-align: right;">(4)</p>
10.2	$A(x) = 6x^2 - 3x^4$ $A'(x) = 12x - 12x^3 = 0$ $12x(1 - x^2) = 0$ $\therefore x \neq 0 \text{ or } x = -1 \text{ or } x = 1$ $\therefore \text{max area: } A(1) = 6(1)^2 - 3(1)^4 = 3 \text{ cm}^2$	$\checkmark 12x - 12x^3 = 0$ $\checkmark \text{values of } x$ $\checkmark \text{correct substitution}$ $\checkmark \text{answer}$ <p style="text-align: right;">(4)</p>
		[8]

QUESTION 7/VRAAG 7

7.1	$f(x) = -2x^2 - 1$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2(x+h)^2 - 1 - (-2x^2 - 1)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 - 1 + 2x^2 + 1}{h}$ $= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= -4x$ OR/OF $f(x+h) = -2(x+h)^2 - 1$ $f(x+h) = -2x^2 - 4xh - 2h^2 - 1$ $f(x+h) - f(x) = -2x^2 - 4xh - 2h^2 - 1 + 2x^2 + 1$ $f(x+h) - f(x) = -4xh - 2h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= -4x$	<p>✓ substitution into the correct formula</p> <p>✓ $-2x^2 - 4xh - 2h^2 - 1$</p> <p>✓ $-4xh - 2h^2$</p> <p>✓ common factor</p> <p>✓ answer (5)</p> <p>OR/OF</p> <p>✓ $-2x^2 - 4xh - 2h^2 - 1$</p> <p>✓ $-4xh - 2h^2$</p> <p>✓ substitution into the correct formula</p> <p>✓ common factor</p> <p>✓ answer (5)</p>
7.2.1	$f(x) = -2x^3 + 3x^2$ $f'(x) = -6x^2 + 6x$	<p>✓ $-6x^2$</p> <p>✓ $+6x$ (2)</p>
7.2.2	$y = 2x + \frac{1}{\sqrt{4x}}$ $y = 2x + \frac{1}{2}x^{-\frac{1}{2}}$ $\frac{dy}{dx} = 2 - \frac{1}{4}x^{-\frac{3}{2}}$	<p>✓ $\frac{1}{2} \checkmark x^{-\frac{1}{2}}$</p> <p>✓ $2 \checkmark -\frac{1}{4}x^{-\frac{3}{2}}$ (4)</p>
7.3	$x < 1$	✓✓ answer (2)
		[13]

QUESTION 8/VRAAG 8

8.1	$y = -10$	✓ answer (1)
8.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f(2) = 2^3 + 4(2)^2 - 7(2) - 10 = 0$	✓ substitution of $x = 2$ ✓ $f(2) = 0$ (2)
8.3	$f(x) = (x-2)(x^2 + 6x + 5)$ $f(x) = (x-2)(x+5)(x+1)$	✓ $(x-2)$ ✓ $(x+5)$ ✓ $(x+1)$ (3)
8.4		✓ x- intercepts ✓ y- intercept ✓ sketching the graph with turning points in 2 nd and 4 th quadrant (3)
8.5.1	$x \in (-3,4; 0,7)$ OR/OF $-3,4 < x < 0,7$	✓✓ $x \in (-3,4; 0,7)$ (2)
8.5.2	$f(x) = x^3 + 4x^2 - 7x - 10$ $f'(x) = 3x^2 + 8x - 7$ $f''(x) = 6x + 8 = 0$ $\therefore x = -\frac{8}{6} = -\frac{4}{3} = -1,33$ OR/OF $\frac{-3,4 + 0,7}{2} = -1,35 = -1,35$	✓ $f''(x) = 6x + 8$ ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
8.5.3	$x \leq -3,4$ or $-1,33 \leq x \leq 0,7$ OR/OF $x \in (-\infty; -3,4] \cup [-1,33; 0,7]$	✓ $x \leq -3,4$ (A) ✓✓ $-1,33 \leq x \leq 0,7$ (A 0,7) (3)
		[16]

QUESTION 9/VRAAG 9

9.1	<p>Perimeter of the square = $12 - 6x$</p> <p>Side length of square = $\frac{12 - 6x}{4} = \frac{6 - 3x}{2} = 3 - \frac{3}{2}x$</p>	<p>✓ $12 - 6x$</p> <p>✓ answer (2)</p>
9.2	<p>$V = \left(\frac{6-3x}{2}\right)^2 (4x)$</p> <p>$= \left(\frac{36 - 36x + 9x^2}{4}\right)(4x)$</p> <p>$= 36x - 36x^2 + 9x^3$</p> <p>$V(x) = 36x - 36x^2 + 9x^3$</p> <p>$V'(x) = 36 - 72x + 27x^2$</p> <p>$36 - 72x + 27x^2 = 0$</p> <p>$9x^2 - 24x + 12 = 0$</p> <p>$3x^2 - 8x + 4 = 0$</p> <p>$(3x - 2)(x - 2) = 0$</p> <p>$x = \frac{2}{3}$ or $x = 2$</p> <p>$V\left(\frac{2}{3}\right) = 36\left(\frac{2}{3}\right) - 36\left(\frac{2}{3}\right)^2 + 9\left(\frac{2}{3}\right)^3$</p> <p>$= \frac{32}{3} \text{ m}^3 = 10,67 \text{ m}^3$</p>	<p>✓ $\left(\frac{6-3x}{2}\right)^2 (4x)$</p> <p>✓ $\left(\frac{36 - 36x + 9x^2}{4}\right)$</p> <p>✓ $36x - 36x^2 + 9x^3$</p> <p>✓ V'</p> <p>✓ $V' = 0$</p> <p>✓ values</p> <p>✓ answer (7)</p>
		[9]

QUESTION/VRAAG 8

8.1	$f(x) = -x^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-(x+h)^2 + x^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-x^2 - 2xh - h^2 + x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-2xh - h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-2x - h)}{h}$ $= \lim_{h \rightarrow 0} (-2x - h)$ $\therefore f'(x) = -2x$ OR/OF $f(x) = -x^2$ $f(x+h) = -(x+h)^2 = -x^2 - 2xh - h^2$ $f(x+h) - f(x) = -x^2 - 2xh - h^2 - (-x^2) = -2xh - h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2xh - h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-2x - h)}{h}$ $= \lim_{h \rightarrow 0} (-2x - h)$ $\therefore f'(x) = -2x$	\checkmark substitution into formula $\checkmark -(x^2 + 2xh + h^2)$ $\checkmark -2xh - h^2$ $\checkmark -2x - h$ \checkmark answer (5)
8.2.1	$f(x) = 4x^3 - 5x^2$ $f'(x) = 12x^2 - 10x$	$\checkmark 12x^2$ (A) $\checkmark -10x$ (A) (2)
8.2.2	$D_x \left[\frac{-6\sqrt[3]{x} + 2}{x^4} \right]$ $= D_x \left[\frac{-6(x)^{\frac{1}{3}}}{x^4} + \frac{2}{x^4} \right]$ $= D_x \left[-6x^{-\frac{11}{3}} + 2x^{-4} \right]$ $= 22x^{-\frac{14}{3}} - 8x^{-5}$	$\checkmark x^{\frac{1}{3}}$ $\checkmark -6x^{-\frac{11}{3}} + 2x^{-4}$ $\checkmark 22x^{-\frac{14}{3}}$ $\checkmark -8x^{-5}$ (4)
		[11]

QUESTION/VRAAG 9

9.1	$f(x) = (x+t)^2(x-3)$ $-3 = (0+t)^2(0-3)$ $1 = t^2$ $t = \pm 1$ $\therefore t = 1$ $f(x) = (x+1)^2(x-3)$ $f(x) = (x^2 + 2x + 1)(x-3)$ $f(x) = x^3 - x^2 - 5x - 3$	$\checkmark f(x) = (x+t)^2(x-3)$ \checkmark subs (0 ; -3) $\checkmark t$ $\checkmark f(x) = (x+1)^2(x-3)$ \checkmark expansion (5)
9.2	$f'(x) = 3x^2 - 2x - 5$ $0 = 3x^2 - 2x - 5$ $0 = (x+1)(3x-5)$ $x = -1$ or $x = \frac{5}{3}$ $N\left(\frac{5}{3}; -\frac{256}{27}\right) = (1,67; -9,48)$	$\checkmark f'(x) = 3x^2 - 2x - 5$ $\checkmark = 0$ \checkmark factors \checkmark x-value ($x > 0$) \checkmark y-value (A) (5)
9.3.1	$x < 3$; $x \neq -1$ OR/OF $x < -1$ or $-1 < x < 3$ OR/OF $(-\infty; -1)$ or $(-1; 3)$	$\checkmark x < 3$ $\checkmark x \neq -1$ (2) OR/OF $\checkmark x < -1$ $\checkmark -1 < x < 3$ (2) OR/OF $\checkmark (-\infty; -1)$ $\checkmark (-1; 3)$ (2)
9.3.2	$x < -1$ or $x > \frac{5}{3}$ OR/OF $x \leq -1$ or $x \geq \frac{5}{3}$ OR/OF $(-\infty; -1)$ or $\left(\frac{5}{3}; \infty\right)$ OR/OF $(-\infty; -1]$ or $\left[\frac{5}{3}; \infty\right)$	$\checkmark x < -1$ $\checkmark x > \frac{5}{3}$ (2) OR/OF $\checkmark (-\infty; -1)$ $\checkmark \left(\frac{5}{3}; \infty\right)$ (2)
9.3.3	$f''(x) > 0$ $6x - 2 > 0$ $x > \frac{1}{3}$ or $\left(\frac{1}{3}; \infty\right)$ OR/OF $\frac{\frac{5}{3} + (-1)}{2} = \frac{1}{3}$ $x > \frac{1}{3}$ or $\left(\frac{1}{3}; \infty\right)$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">ANSWER ONLY: FULL MARKS</div> $\checkmark 6x - 2$ $\checkmark \frac{1}{3}$ $\checkmark x > \frac{1}{3}$ (3) OR/OF \checkmark substitution $\checkmark \frac{1}{3}$ $\checkmark x > \frac{1}{3}$ (3)

9.4	$\text{Distance} = x^3 - x^2 - 5x - 3 - (3x^2 - 2x - 5)$ $= x^3 - 4x^2 - 3x + 2$ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ $0 = 3x^2 - 8x - 3$ $0 = (3x + 1)(x - 3)$ $x = 3 \text{ or } x = -\frac{1}{3}$ <p>Max distance</p> $= \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 2$ $= \frac{68}{27} = 2,52$	$\checkmark x^3 - 4x^2 - 3x + 2$ $\checkmark \frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ \checkmark factors \checkmark x-values $\checkmark x = -\frac{1}{3}$ \checkmark answer <div style="text-align: right;">(6)</div>
		[23]

Telbeginsel en Waarskynlikheid

Mei/Junie 2024

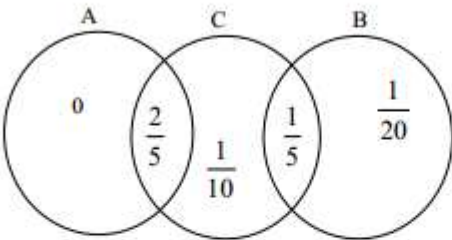
QUESTION 11/VRAAG 11

11.1	$P(A) + P(B) = 0,52$ $0,4 + P(B) = 0,52$ $P(B) = 0,12$	\checkmark substitution \checkmark answer <div style="text-align: right;">(2)</div>
11.2.1	$P(\text{sandwich}) = \frac{4}{25}$ <p style="text-align: center;">OR/OF</p> $0,02 + 0,01 + 0,04 + 0,09 = \frac{4}{25} = 0,16$	\checkmark answer <div style="text-align: right;">(1)</div> <p style="text-align: center;">OR/OF</p> \checkmark answer <div style="text-align: right;">(1)</div>
11.2.2	$P(\text{at least two events}) = 0,02 + 0,01 + 0,03 + 0,04$ $= 0,1$	$\checkmark 0,02 + 0,01 + 0,03 + 0,04$ \checkmark answer <div style="text-align: right;">(2)</div>
11.2.3	$P(\text{not any}) = 1 - (0,1 + 0,04 + 0,09 + 0,2)$ $= 0,57$	$\checkmark 1 - (0,1 + 0,04 + 0,09 + 0,2)$ \checkmark answer <div style="text-align: right;">(2)</div>
11.3.1	$7! = 5040$	$\checkmark 7!$ <div style="text-align: right;">(1)</div>
11.3.2	$P(4 \text{ players alphabetically}) = \frac{1}{7 \times 6 \times 5 \times 4} = \frac{1}{840}$	$\checkmark 1$ $\checkmark 840$ $\checkmark \frac{1}{840}$ <div style="text-align: right;">(3)</div>

11.3.3	<table><tr><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td></tr></table> <p>F arrangements: 4!</p> <p>M arrangements: 5 options with 3 males = $5 \times 4 \times 3$</p> <p>$4! \times 5 \times 4 \times 3$</p> <p>= 1 440</p> <p>OR/OF</p> <p>10 Options:</p> <p>F M F M F M F</p> <p>M F M F M F F</p> <p>F F M F M F M</p> <p>F M F M F F M</p> <p>M F M F F M F</p> <p>M F M F F F M</p> <p>F M F F M F M</p> <p>M F F M F F M</p> <p>M F F M F M F</p> <p>M F F F M F M</p> <p>Hence $10 \times 4! \times 3! = 1440$</p>		F		F		F		F		<p>✓4!</p> <p>✓$5 \times 4 \times 3$</p> <p>✓1 440</p> <p>(3)</p> <p>OR/OF</p> <p>✓$4! \times 3!$</p> <p>✓$\times 10$</p> <p>✓1 440</p> <p>(3)</p>
	F		F		F		F				
		[14]									
	TOTAL/TOTAAL: 150										

QUESTION 10/VRAAG 10

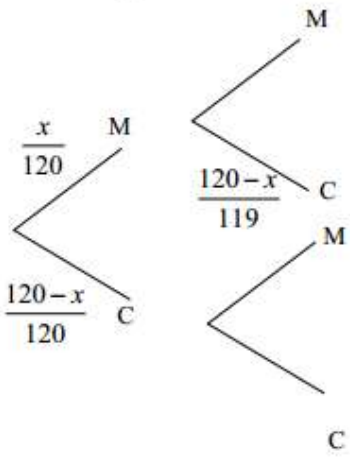
10.1.1	<p>Event A</p> <p>Event B</p> <p>HC</p> <p>HN</p> <p>SC</p> <p>SN</p>	<p>✓ Event A</p> <p>✓ Event B Medication: for $P(C) = \frac{3}{5}$</p> <p>✓ Event B sugar pill: for $P(NC) = \frac{7}{10}$</p> <p>(3)</p>
10.1.2	<p>$P(\text{Not Cured}) = P(H) \times P(NC) + P(S) \times P(NC)$</p> <p>$= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)$</p> <p>$= \frac{11}{20} = 0,55$</p>	<p>✓ substitution</p> <p>✓ answer</p> <p>(2)</p>
10.2.1	<p>$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$</p> <p>$P(A \text{ and } B) = \frac{13}{20} - \frac{2}{5} - \frac{1}{4} = 0$</p> <p>Events are mutually exclusive</p> <p>OR/OF</p> <p>$P(A) + P(B) = \frac{2}{5} + \frac{1}{4}$</p> <p>$= \frac{13}{20}$</p> <p>$P(A \text{ or } B) = P(A) + P(B)$</p> <p>$P(A \text{ and } B) = 0$</p> <p>Events are mutually exclusive</p>	<p>✓ substitution</p> <p>✓ answer ($P(A \text{ and } B) = 0$)</p> <p>(2)</p> <p>OR/OF</p> <p>✓ substitution</p> <p>✓ answer ($P(A \text{ and } B) = 0$)</p> <p>(2)</p>

10.2.2	$P(B \text{ and } C) = \frac{1}{5} = 0,2$ $P(\text{ only } C) = \frac{7}{10} - \frac{2}{5} - \frac{1}{5} = \frac{1}{10} = 0,1$ 	$\checkmark P(B \text{ and } C) = \frac{1}{5} \text{ (A)}$ $\checkmark \frac{7}{10} - \frac{2}{5} - \frac{1}{5}$ $\checkmark \frac{1}{10}$	(3)	
10.2.3	$P(\text{no event}) = 1 - \left(\frac{2}{5} + \frac{1}{10} + \frac{1}{5} + \frac{1}{20} \right) = \frac{1}{4} = 0,25$	$\checkmark 1 - (P(A) \text{ or } P(B) \text{ or } P(C))$ $\checkmark \text{answer}$	(2)	
10.3.1	$3! \times 5!$ $= 720$	<div>Answer only:</div> <div>Full Marks</div>	$\checkmark 3!$ $\checkmark 3! \times 5! \text{ (A)}$	(2)
10.3.2	$\frac{7! - 6! \times 2}{7!} = \frac{5}{7} = 0,71$ <p>OR/OF</p> $1 - \frac{6! \times 2}{7!}$ $= 1 - \frac{2}{7}$ $= \frac{5}{7} = 0,71$	$\checkmark 7! - 6! \times 2$ $\checkmark \text{denominator } (7!)$ $\checkmark \text{answer}$ <p>OR/OF</p> $\checkmark 6! \times 2$ $\checkmark \text{denominator } (7!)$		(3)
				[17]

Mei/Junie 2022

QUESTION/VRAAG 10

10.1.1	$7! = 5\,040$	$\checkmark \checkmark \text{ answer}$ (2)
10.1.2	$4! \times 4!$ $= 576$ $P(\text{African flags together}) = \frac{576}{5040} \left(= \frac{4}{35} = 0,11 \right)$	$\checkmark 4!$ $\checkmark 4! \times 4!$ $\checkmark \text{answer (A)}$ (3)
10.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,88 = 0,4 + P(B) - P(A \text{ and } B)$ $0,88 = 0,4 + P(B) - 0,4P(B)$ $0,48 = 0,6P(B)$ $P(B) = 0,8$	$\checkmark \text{subs into rule}$ $\checkmark P(A \text{ and } B) = 0,4P(B)$ $\checkmark \text{answer}$ (3)

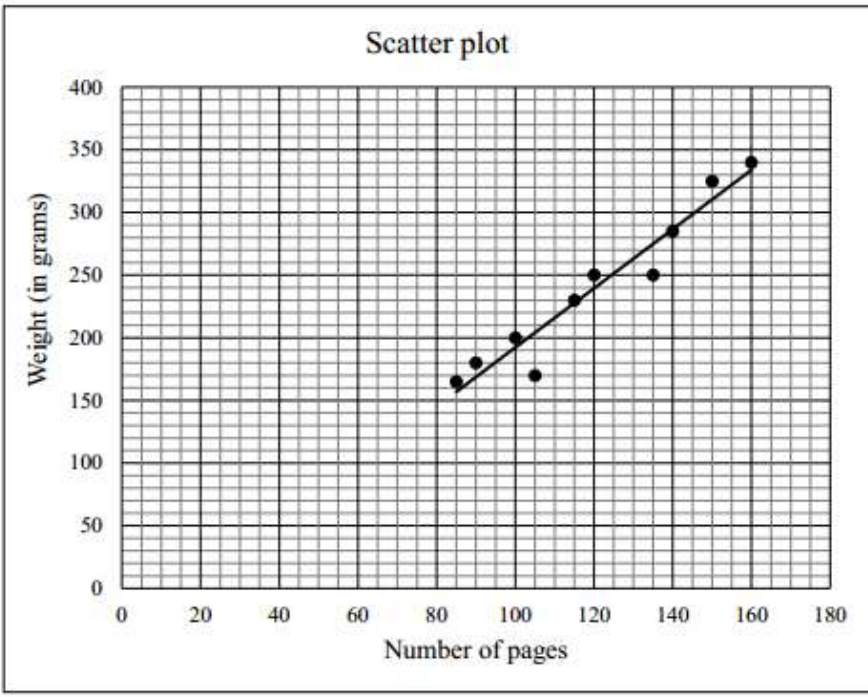
10.3	<div>First Passenger</div> <div>Second Passenger</div> <div></div>	
	<div>Probability of first passenger choosing meat = $\frac{x}{120}$</div> <div>Probability of second passenger choosing cheese = $\frac{120-x}{119}$</div> <div>$\frac{x}{120} \times \frac{120-x}{119} = \frac{18}{85}$</div> <div>$120x - x^2 = 3\,024$</div> <div>$x^2 - 120x + 3\,024 = 0$</div> <div>$(x - 84)(x - 36) = 0$</div> <div>$x = 84 \quad \text{or} \quad x = 36$</div> <div>$\therefore P(\text{1}^{\text{st}} \text{ cheese}) = \frac{36}{120} = \frac{3}{10}$</div>	<div>$\checkmark \frac{x}{120}$</div> <div>$\checkmark \frac{120-x}{119}$</div> <div>$\checkmark \frac{x}{120} \times \frac{120-x}{119} = \frac{18}{85}$</div> <div>$\checkmark x = 84 \quad \text{or} \quad x = 36$</div> <div>$\checkmark \frac{3}{10}$</div> <div>(5)</div>
		[13]

VRAESTEL 2

Statistiek en Regressie

Mei/Junie 2024

QUESTION/VRAAG 1

1.1	$a = -43,72$ $b = 2,36$ $y = -43,72 + 2,36x$	✓ $a = -43,72$ ✓ $b = 2,36$ ✓ equation (3)
1.2	<p style="text-align: center;">Scatter plot</p>  <p>The scatter plot displays a positive linear relationship between the number of pages and weight. The y-axis is labeled 'Weight (in grams)' and ranges from 0 to 400 in increments of 50. The x-axis is labeled 'Number of pages' and ranges from 0 to 180 in increments of 20. There are 12 data points plotted, and a straight line of best fit is drawn through them, starting at approximately (85, 160) and ending at (160, 340).</p>	✓ any correct two points ✓ straight line joining the points for $x \in [85 ; 160]$ (2)
1.3	$y = -43,72 + 2,36(110)$ $y = 215,88$ OR $y = 215,90$ (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)

1.4	$y = -43,72 + 2,36(130)$ $y = 263,08$ $\text{Percentage increase in weight} = \frac{263,08 - 215,88}{215,88} \times 100$ $= 21,86\%$ OR $y = 263,08$ $\text{Percentage} = \frac{263,08}{215,88} \times 100$ $= 121,86\%$ $\text{Percentage increase in weight} = 121,86 - 100 = 21,86$	✓ y -value ✓ difference between y-values ✓ +ve answer (3) ✓ y -value ✓ difference between % ✓ +ve answer (3)
		[10]

QUESTION/VRAAG 2

2.1	<table border="1"> <thead> <tr> <th>Distance (x km)</th><th>Frequency</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td>$0 \leq x < 5$</td><td>3</td><td>3</td></tr> <tr> <td>$5 \leq x < 10$</td><td>7</td><td>10</td></tr> <tr> <td>$10 \leq x < 15$</td><td>20</td><td>30</td></tr> <tr> <td>$15 \leq x < 20$</td><td>12</td><td>42</td></tr> <tr> <td>$20 \leq x < 25$</td><td>5</td><td>47</td></tr> <tr> <td>$25 \leq x < 30$</td><td>3</td><td>50</td></tr> </tbody> </table>	Distance (x km)	Frequency	Cumulative frequency	$0 \leq x < 5$	3	3	$5 \leq x < 10$	7	10	$10 \leq x < 15$	20	30	$15 \leq x < 20$	12	42	$20 \leq x < 25$	5	47	$25 \leq x < 30$	3	50	✓ 10 ✓ all values correct (2)
Distance (x km)	Frequency	Cumulative frequency																					
$0 \leq x < 5$	3	3																					
$5 \leq x < 10$	7	10																					
$10 \leq x < 15$	20	30																					
$15 \leq x < 20$	12	42																					
$20 \leq x < 25$	5	47																					
$25 \leq x < 30$	3	50																					
2.2	<p style="text-align: center;"><i>Ogive/Ogief</i></p>	✓ grounding ✓ plotting a min of 3 points (cf at upper limits) ✓ smooth, increasing curve (3)																					

2.3	$Q_3 = 17,8$ $Q_1 = 11$ $IQR = 6,8$	✓ Q_3 (accept between 17-19) and Q_1 (accept between 10-12,5) ✓ answer (accept 5-9) (2)
2.4	$5 \leq x < 10$	✓ $5 \leq x < 10$ (1)
2.5	Estimated mean = $\frac{2,5(3) + 7,5(11) + 12,5(20) + 17,5(8) + 22,5(5) + 27,5(3)}{50}$ $= \frac{675}{50}$ $= 13,5 \text{ km}$	✓ new frequencies ✓ $\sum fx$ ✓ answer (3)
		[11]

Mei/Junie 2023

QUESTION/VRAAG 1

1.1.1	$a = 1730,22$ $b = 13,96$ $\hat{y} = 1730,22 + 13,96x$	✓ $a = 1730,22$ ✓ $b = 13,96$ ✓ equation (3)
1.1.2	$\hat{y} = 1730,22 + 13,96x$ $\hat{y} = 1730,22 + 13,96(28\,500)$ $\hat{y} = R399\,590,22$ OR/OF $\hat{y} = R399\,599,64 \text{ (calc)}$	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.1.3	$r = 0,98002 \dots$ $r = 0,98$	✓ answer (1)
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. / <i>Daar is 'n baie sterk positiewe korrelasie tussen die bedrag spandeer op advertensie en die verkope.</i>	✓ strong positive (1)
1.2.1	$\bar{x} = \frac{1\,552\,195}{9}$ $\bar{x} = 172\,466,11$	✓ $\bar{x} = \frac{1\,552\,195}{9}$ ✓ answer (2)
1.2.2	$\sigma = 56\,950,09$	✓ answer (1)
1.2.3	$\bar{x} + \sigma$ $= 172\,466,11 + 56\,950,09$ $= 229\,416,20$ 2 years/jaar	✓ $\bar{x} + \sigma$ ✓ answer (2)
		[12]

QUESTION/VRAAG 2

2.1	$35 < x \leq 45$	✓ answer (1)																								
2.2	320 people/mense	✓ answer (1)																								
2.3	<table border="1"> <thead> <tr> <th>AGE</th><th>NUMBER OF PEOPLE</th><th>CUMULATIVE FREQUENCY</th></tr> </thead> <tbody> <tr> <td>$5 < x \leq 15$</td><td>20</td><td>20</td></tr> <tr> <td>$15 < x \leq 25$</td><td>25</td><td>45</td></tr> <tr> <td>$25 < x \leq 35$</td><td>60</td><td>105</td></tr> <tr> <td>$35 < x \leq 45$</td><td>90</td><td>195</td></tr> <tr> <td>$45 < x \leq 55$</td><td>55</td><td>250</td></tr> <tr> <td>$55 < x \leq 65$</td><td>40</td><td>290</td></tr> <tr> <td>$65 < x \leq 75$</td><td>30</td><td>320</td></tr> </tbody> </table> <div> <p style="text-align: center;">OGIVE/OGIEF</p> </div>	AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY	$5 < x \leq 15$	20	20	$15 < x \leq 25$	25	45	$25 < x \leq 35$	60	105	$35 < x \leq 45$	90	195	$45 < x \leq 55$	55	250	$55 < x \leq 65$	40	290	$65 < x \leq 75$	30	320	<p>✓ cumulative frequency</p> <p>✓ grounding</p> <p>✓ plotting at upper limit</p> <p>✓ shape</p> <p>(4)</p>
AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY																								
$5 < x \leq 15$	20	20																								
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$35 < x \leq 45$	90	195																								
$45 < x \leq 55$	55	250																								
$55 < x \leq 65$	40	290																								
$65 < x \leq 75$	30	320																								
2.4	Median = 41	✓✓ answer (2)																								
		[8]																								

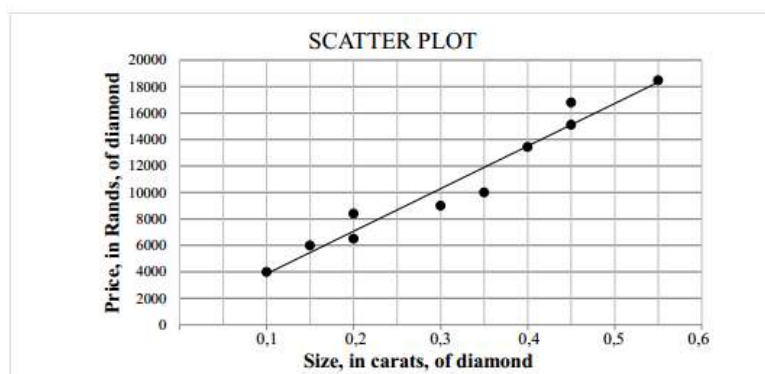
QUESTION/VRAAG 1

1.1	Modal class: $9 < m \leq 11$	✓ answer (1)																								
1.2	<table border="1"> <thead> <tr> <th>Mass (in kg)</th><th>Frequency</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td>$5 < m \leq 7$</td><td>6</td><td>6</td></tr> <tr> <td>$7 < m \leq 9$</td><td>18</td><td>24</td></tr> <tr> <td>$9 < m \leq 11$</td><td>21</td><td>45</td></tr> <tr> <td>$11 < m \leq 13$</td><td>19</td><td>64</td></tr> <tr> <td>$13 < m \leq 15$</td><td>11</td><td>75</td></tr> <tr> <td>$15 < m \leq 17$</td><td>4</td><td>79</td></tr> <tr> <td>$17 < m \leq 19$</td><td>1</td><td>80</td></tr> </tbody> </table>	Mass (in kg)	Frequency	Cumulative frequency	$5 < m \leq 7$	6	6	$7 < m \leq 9$	18	24	$9 < m \leq 11$	21	45	$11 < m \leq 13$	19	64	$13 < m \leq 15$	11	75	$15 < m \leq 17$	4	79	$17 < m \leq 19$	1	80	✓ adding ✓ 80 (2)
Mass (in kg)	Frequency	Cumulative frequency																								
$5 < m \leq 7$	6	6																								
$7 < m \leq 9$	18	24																								
$9 < m \leq 11$	21	45																								
$11 < m \leq 13$	19	64																								
$13 < m \leq 15$	11	75																								
$15 < m \leq 17$	4	79																								
$17 < m \leq 19$	1	80																								
1.3		✓ grounding (5 ; 0) ✓ points ✓ shape (3)																								
1.4	Median mass: 10,5 kg	✓✓ answer (2)																								
1.5.1	$\bar{x} = \frac{(6 \times 6 + 18 \times 8 + 21 \times 10 + 19 \times 12 + 11 \times 14 + 4 \times 16 + 1 \times 18)}{80}$ $= \frac{854}{80}$ $= 10,68$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 2/2</div>	✓ 854 ✓ answer (2)																								
1.5.2	Learners' bags are heavier than the stipulated international guideline. Estimated mean = 10,68 kg 10% of 80 kg = 8 kg 10,68 kg > 8 kg	✓ answer ✓ 8 kg (2)																								

<p>OR/ OF</p> <p>Learners' bags are heavier than the stipulated international guideline.</p> <p>Estimated mean $= \frac{10,68}{80} \times 100$</p> <p>$= 13,35\%$</p> <p>$13,35\% > 10\%$</p>	<p>✓ answer</p> <p>✓ 13,35%</p> <p>(2)</p>
[12]	

QUESTION/VRAAG 2

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480

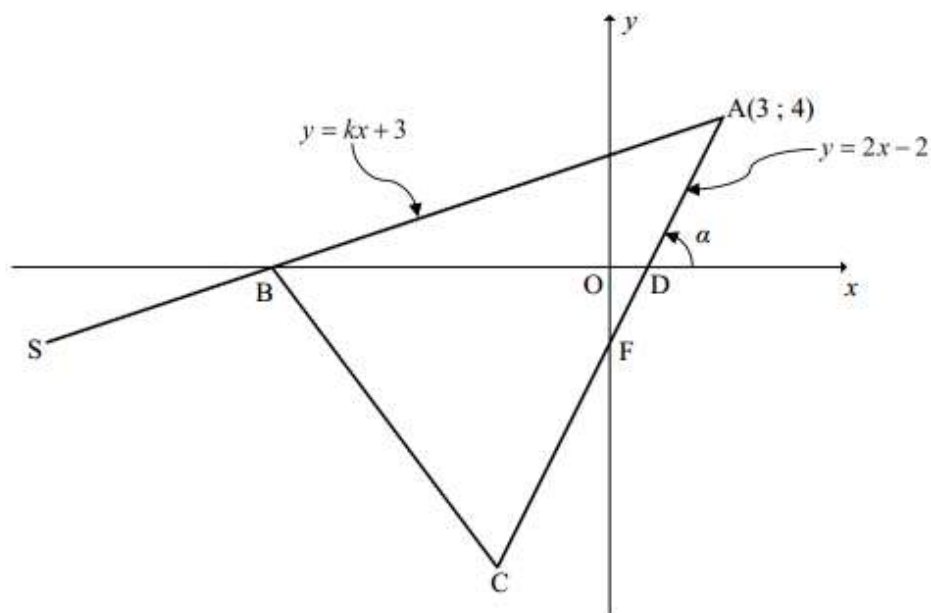


2.1	$a = 634,382\dots$ $b = 32\,189,263\dots$ $\hat{y} = 634,38 + 32189,26x$	<p>✓ a</p> <p>✓ b</p> <p>✓ equation</p> <p>(3)</p>
2.2	<p>Answer only 3/3</p> <p>$\hat{y} = 634,38 + 32189,26(0,25)$</p> <p>$= R8\,681,70$</p> <p>OR/OF</p> <p>$\hat{y} = R8\,681,70$ (if using calculator)</p>	<p>✓ substitution</p> <p>✓ answer</p> <p>(2)</p> <p>✓ ✓ answer</p> <p>(2)</p>
2.3	<p>Average price increase $= R \frac{32189,26}{20}$ per 0,05 carat</p> <p>$= R1\,609,46$ per 0,05 carat</p> <p>OR/OF</p> <p>Average price increase $= 0,05 \times 32189,26$</p> <p>$= R1\,609,46$ per 0,05 carat</p> <p>OR/OF</p> <p>at 0,3: $\hat{y} = R10\,291,16$</p> <p>\therefore Average price increase $= 10\,291,16 - 8\,681,70$</p> <p>$= R1\,609,46$ per 0,05 carat</p> <p>Answer only 2/2</p>	<p>✓ divide gradient by 20</p> <p>✓ answer</p> <p>(2)</p> <p>✓ multiply gradient by 0,05</p> <p>✓ answer</p> <p>(2)</p> <p>✓ Estimated price of a 0,3 carat diamond</p> <p>✓ answer</p> <p>(2)</p>
2.4	<p>The point (0,35 ; 11500) is closer to the least squares regression line.</p>	<p>✓ reason</p> <p>(1)</p>
[8]		

Analitiese Meetkunde

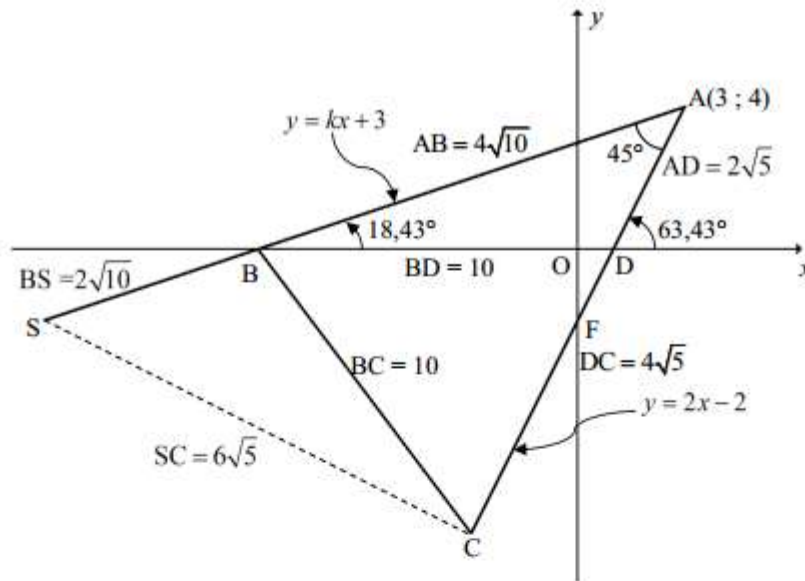
Mei/Junie 2024

QUESTION/VRAAG 3



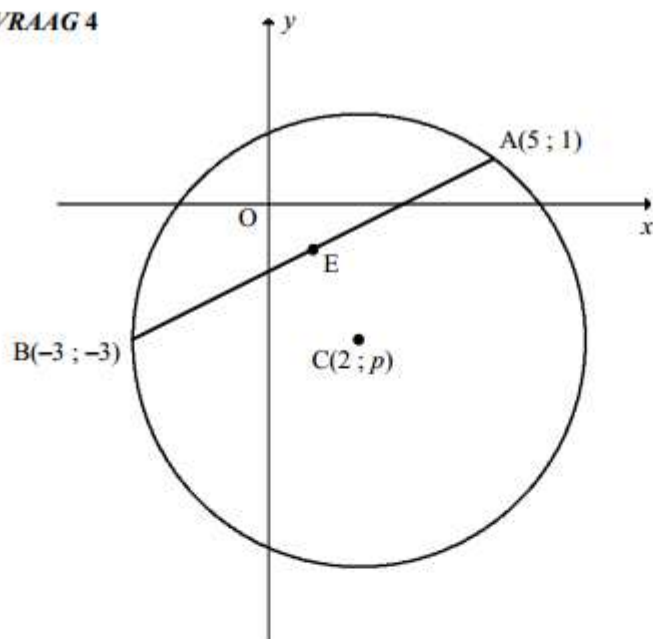
3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$ OR y-intercept of AB: (0 ; 3) $m_{AB} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4) ✓ substitution (0 ; 3)	(1) (1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ B(-9 ; 0)	✓ $y = 0$ ✓ answer	 (2)

3.3	$F(0; -2)$ $F\left(\frac{x+3}{2}; \frac{y+4}{2}\right)$ $\frac{x+3}{2} = 0 \quad \frac{y+4}{2} = -2$ $x = -3 \quad y = -8$ $C(-3; -8)$ OR by translation $F(0; -2)$ $A \rightarrow F(x; y) \rightarrow (x-3; y-6)$ $F \rightarrow C(0; -2) \rightarrow (0-3; -2-6) = (-3; -8)$	$\checkmark F(0; -2)$ $\checkmark \frac{x+3}{2} = 0; \frac{y+4}{2} = -2$ $\checkmark x\text{-value} \quad \checkmark y\text{-value}$ (4) $\checkmark F(0; -2)$ $\checkmark (x-3; y-6)$ $\checkmark x\text{-value} \quad \checkmark y\text{-value}$ (4)
3.4	$m_{BC} = \frac{0 - (-8)}{-9 - (-3)} \quad \text{OR} \quad m_{BC} = \frac{-8 - 0}{-3 - (-9)}$ $m_{BC} = -\frac{4}{3}$ $y = -\frac{4}{3}x + c$ $(-2) = -\frac{4}{3}(-15) + c$ $c = -22$ $y = -\frac{4}{3}x - 22$ OR $m_{BC} = \frac{0 - (-8)}{-9 - (-3)} \quad \text{OR} \quad m_{BC} = \frac{-8 - 0}{-3 - (-9)}$ $m_{BC} = -\frac{4}{3}$ $y - y_1 = -\frac{4}{3}(x - x_1)$ $y - (-2) = -\frac{4}{3}(x - (-15))$ $y + 2 = -\frac{4}{3}x - 20$ $y = -\frac{4}{3}x - 22$	\checkmark substitution of B and C into the gradient formula $\checkmark m_{BC}$ $\checkmark m_{\text{line}} = m_{BC}$ \checkmark substitution of S(-15; -2) \checkmark equation (5) \checkmark substitution into the gradient formula $\checkmark m_{BC}$ $\checkmark m_{\text{line}} = m_{BC}$ \checkmark substitution of S(-15; -2) \checkmark equation (5)



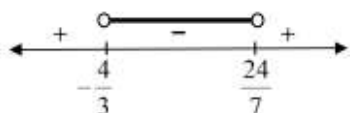
3.5	$\tan \alpha = m_{AC} = 2$ $\alpha = 63,43^\circ$ $\tan \hat{A}BD = m_{AS} = \frac{1}{3}$ $\hat{A}BD = 18,43^\circ$ $\hat{B}AC = \alpha - \hat{A}BD$ $\hat{B}AC = 63,43^\circ - 18,43^\circ$ $\hat{B}AC = 45^\circ$ OR $AB = \sqrt{(-9-3)^2 + (0-4)^2}$ $AB = 4\sqrt{10}$ $BD = 10$ $AD = \sqrt{(3-1)^2 + (4-0)^2}$ $AD = 2\sqrt{5}$ $BD^2 = AB^2 + AD^2 - 2AB \cdot AD \cos \hat{B}AC$ $(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5}) \cos \hat{B}AC$ $\cos \hat{B}AC = \frac{\sqrt{2}}{2}$ $\hat{B}AC = 45^\circ$	$\checkmark \tan \alpha = m_{AC} = 2$ $\checkmark \alpha = 63,43^\circ$ $\checkmark \tan \hat{A}BD = m_{AS} = \frac{1}{3}$ $\checkmark \hat{A}BD = 18,43^\circ$ \checkmark answer \checkmark length of AB \checkmark calculation of remaining 2 lengths \checkmark substitution into cosine-rule \checkmark rewriting in terms of $\cos \hat{B}AC$ \checkmark answer (5)
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QUESTION/VRAAG 4



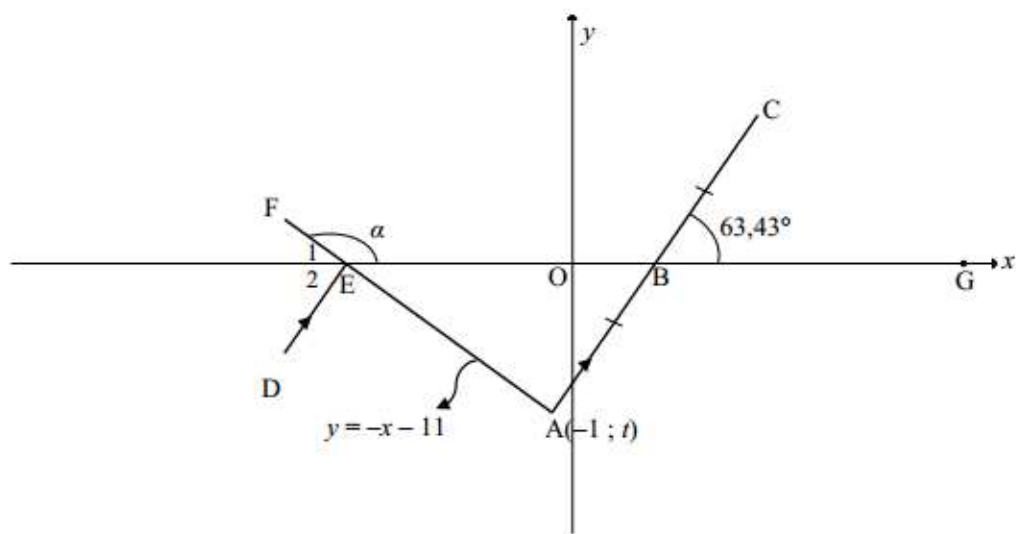
4.1	$E\left(\frac{5+(-3)}{2}; \frac{1+(-3)}{2}\right)$ $\therefore E(1; -1)$	$\checkmark x=1 \quad \checkmark y=-1$ (2)
4.2	$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$ $AB = \sqrt{(5 - (-3))^2 + (1 - (-3))^2}$ $AB = \sqrt{80} = 4\sqrt{5} = 8,94 \text{ units}$	$\checkmark AB = \sqrt{80} = 4\sqrt{5} = 8,94$ (1)
4.3	$m_{AB} = \frac{1 - (-3)}{5 - (-3)}$ $m_{AB} = \frac{1}{2}$ $\therefore m_{CE} = -2 \quad [CE \perp AB]$ $E(1; -1)$ $y = -2x + c \quad \text{OR} \quad y - y_1 = -2(x - x_1)$ $(-1) = -2(1) + c \quad y - (-1) = -2(x - 1)$ $c = 1 \quad y = -2x + 1$	$\checkmark m_{AB} = \frac{1}{2}$ $\checkmark m_{CE}$ \checkmark substitution of E \checkmark equation (4)

4.4	$y = -2x + 1$ $p = -2(2) + 1$ $p = -3$ OR $m_{CE} = -2$ $\frac{p - (-1)}{2 - 1} = -2$ $p + 1 = -2$ $p = -3$	✓ substitution of $C(2; p)$ into \perp bisector of AB ✓ substitution of C and E into the gradient formula
4.5	$BC = r = 5$ units $\therefore (x - 2)^2 + (y + 3)^2 = 25$ $x^2 - 4x + 4 + y^2 + 6y + 9 = 25$ $x^2 + y^2 - 4x + 6y - 12 = 0$	✓ $BC = r = 5$ units ✓ $(x - 2)^2 + (y + 3)^2 = r^2$ ✓ $x^2 - 4x + 4 + y^2 + 6y + 9 = 25$

4.6	$(x - 2)^2 + (y + 3)^2 = 25$ $y = tx + 8$ $(x - 2)^2 + (tx + 8 + 3)^2 = 25$ $x^2 - 4x + 4 + t^2x^2 + 22tx + 121 - 25 = 0$ $x^2(t^2 + 1) + x(22t - 4) + 100 = 0$ $\Delta < 0$ $(22t - 4)^2 - 4(t^2 + 1)(100) < 0$ $484t^2 - 176t + 16 - 400t^2 - 400 < 0$ $84t^2 - 176t - 384 < 0$ $21t^2 - 44t - 96 < 0$ $(7t - 24)(3t + 4) < 0$ CV: $\frac{24}{7}; -\frac{4}{3}$  $\therefore t \in \left(-\frac{4}{3}; \frac{24}{7}\right)$ OR $-\frac{4}{3} < t < \frac{24}{7}$	✓ substitution of $y = tx + 8$ ✓ standard form ✓ $\Delta < 0$ ✓ standard form of Δ ✓ critical values ✓ answer
		(6)

[18]

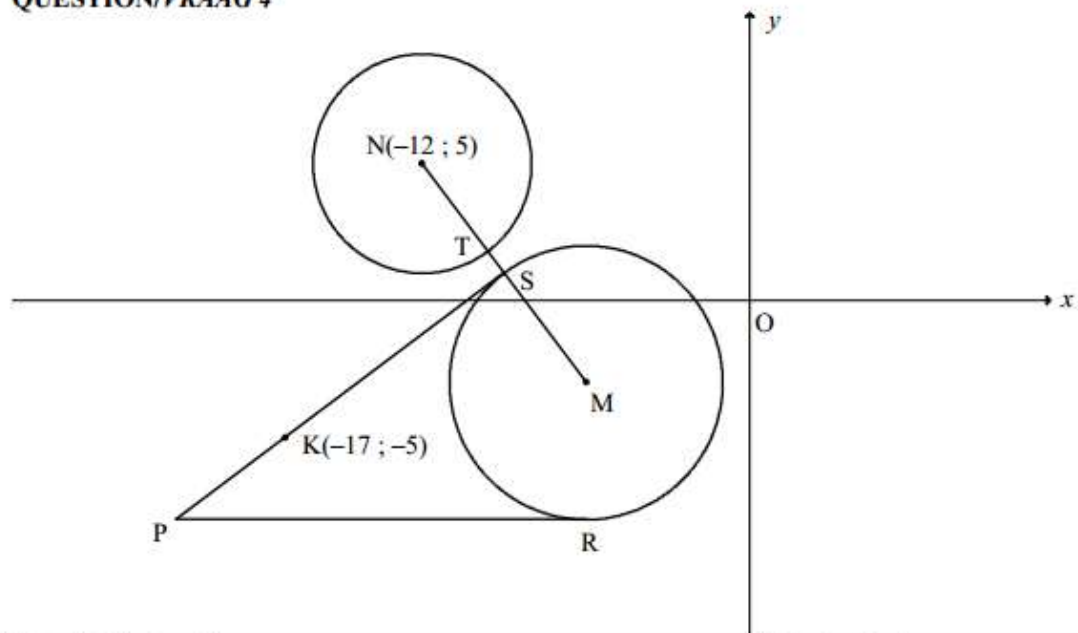
QUESTION/VRAAG 3



3.1.1	$y = -x - 11$ $A(-1; t)$ $t = -(-1) - 11$ $t = -10$	✓ substitution ✓ value of t (2)
3.1.2	$\tan \alpha = -1$ $\text{ref. } \angle = 45^\circ$ $\therefore \alpha = 135^\circ$	✓ $\tan \alpha = -1$ ✓ 135° (2)
3.1.3	$\tan 63,43^\circ = m_{AC}$ $m_{AC} = 2$	✓ $\tan 63,43^\circ = m_{AC}$ ✓ answer (2)
3.2	$m_{AC} = 2$ $A(-1; -10)$ $y = 2x + k$ $-10 = 2(-1) + k$ $k = -8$ $y = 2x - 8$	<p>OR/OF $y - y_1 = 2(x - x_1)$ $y - (-10) = 2(x - (-1))$ $y = 2x - 8$</p> ✓ substitution of m and A ✓ equation (2)

3.3.1	$y = 2x - 8$ $0 = 2x - 8$ $x_B = 4$ $\frac{x_C + (-1)}{2} = 4$ $x_C = 9$ $\frac{y_C + (-10)}{2} = 0$ $y_C = 10$ OR/OF by translation / <i>met translasië</i> $A \rightarrow B(x; y) \rightarrow (x + 5; y + 10)$ $B \rightarrow C(4; 0) \rightarrow (4 + 5; 0 + 10) = (9; 10)$	$\checkmark x_B = 4$ $\checkmark x_C = 9 \quad \checkmark y_C = 10$ (3) $\checkmark (x + 5; y + 10)$ $\checkmark x_C = 9 \quad \checkmark y_C = 10$ (3)
3.3.2	$\hat{A}BE = 63,43^\circ$ $\hat{E}_2 = 63,43^\circ$ $\hat{E}_1 = 45^\circ$ $\hat{F}ED = 108,43^\circ$ OR/OF $\hat{E}AB = 135^\circ - 63,43^\circ$ $\hat{E}AB = 71,57^\circ$ $\hat{D}EA = \hat{E}AB = 71,57^\circ$ $\hat{F}ED = 108,43^\circ$ OR/OF $\hat{A}BE = 63,43^\circ$ $\hat{D}EO = 116,57^\circ$ $\hat{F}ED = 360^\circ - (116,57^\circ + 135^\circ)$ $= 108,43^\circ$	[vert. opp \angle 's =] [corres. \angle 's, DE \parallel AB] [\angle s on a str line] $\checkmark \hat{A}BE = 63,43^\circ$ $\checkmark \hat{E}_1 = 45^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3) $\checkmark \hat{E}AB = 71,57^\circ$ $\checkmark \hat{D}EA = \hat{E}AB = 71,57^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3) $\checkmark \hat{A}BE = 63,43^\circ$ $\checkmark \hat{D}EO = 116,57^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3)
3.4	$y = 0$ $x_E = -11$ $\frac{x_G + (-11)}{2} = 4$ $x_G = 19$ $(x - 19)^2 + y^2 = 15^2$ $(x - 19)^2 + y^2 = 225$	$\checkmark x_E = -11$ $\checkmark x_G = 19$ $\checkmark (x - 19)^2 + y^2 = 225$ (4)

QUESTION/VRAAG 4



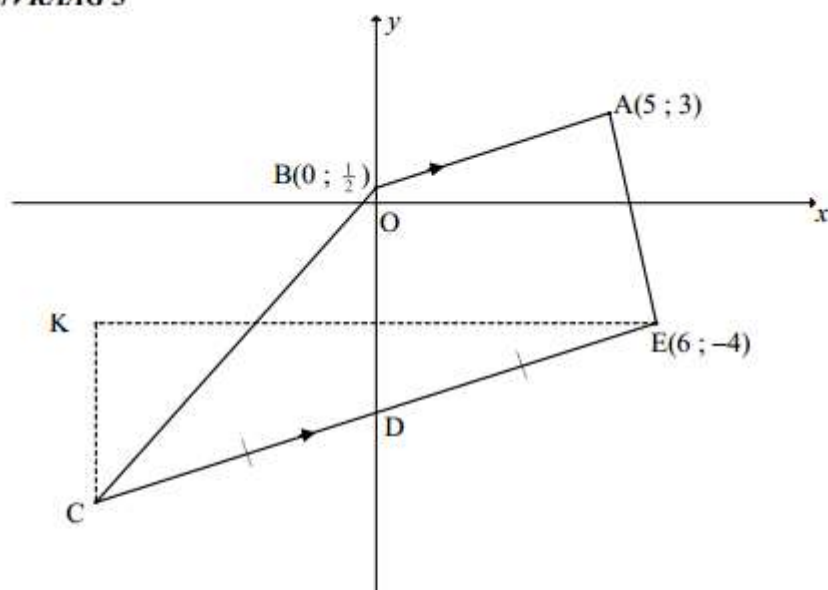
4.1	$M(-6; -3)$	✓ -6 ✓ -3 (2)
4.2.1	$x^2 + y^2 + 24x - 10y + 153 = 0$ $(x+12)^2 + (y-5)^2 = -153 + 144 + 25$ $(x+12)^2 + (y-5)^2 = 16$ $r^2 = 16$ $r = 4$ units	✓ $r^2 = -153 + 144 + 25$ ✓ length of radius (2)
4.2.2	$NM = \sqrt{(-12 - (-6))^2 + (5 - (-3))^2}$ $NM = 10$ units $SM = 5$ units $\therefore TS = 10 - 5 - 4 = 1$ unit	✓ substitution into distance formula ✓ $NM = 10$ units ✓ $SM = 5$ units ✓ answer (4)
4.3.1	$R(-6; -8)$ $y = -8$	✓ $y_R = -8$ ✓ answer (2)

4.3.2	$m_{NM} = \frac{5 - (-3)}{-12 - (-6)}$ $m_{NM} = -\frac{4}{3}$ $m_{\text{tangent}} = \frac{3}{4}$ $-5 = \frac{3}{4}(-17) + c \quad \text{OR/OF} \quad y - y_1 = \frac{3}{4}(x - x_1)$ $c = \frac{31}{4} \quad y - (-5) = \frac{3}{4}(x - (-17))$ $y = \frac{3}{4}x + \frac{31}{4} \quad y = \frac{3}{4}x + \frac{31}{4}$ <p>OR/OF</p> $NS = SM = 5$ $S\left(\frac{-12-6}{2}; \frac{5-3}{2}\right)$ $S(-9; 1)$ $m_{SK} = \frac{1 - (-5)}{-9 + 17}$ $= \frac{6}{8} = \frac{3}{4}$ $y + 5 = \frac{3}{4}(x + 17)$ $y = \frac{3}{4}x + \frac{31}{4} \quad \text{or} \quad y = \frac{3}{4}x + 7\frac{3}{4}$	<p>✓ substitution</p> <p>✓ $m_{NM} = -\frac{4}{3}$</p> <p>✓ $m_{\text{tangent}} = \frac{3}{4}$</p> <p>✓ substitution of m and N</p> <p>✓ equation (5)</p> <p>✓ S midpoint</p> <p>✓ coordinates of S</p> <p>✓ $m_{\text{tangent}} = \frac{3}{4}$</p> <p>✓ substitution of m and $K(-17; -5)$ or S</p> <p>✓ equation (5)</p>
4.4.1	$-8 = \frac{3}{4}x + \frac{31}{4}$ $-32 = 3x + 31$ $3x = -63$ $x = -21$ $P(-21; -8)$ $R(-6; -8)$ <p>$PR = PS = 15$ units [tangents from same point]</p> <p>$MS = MR = 5$ units</p> <p>Perimeter $PSMR = 15 + 15 + 5 + 5$ $= 40$ units</p>	<p>✓ $-8 = \frac{3}{4}x + \frac{31}{4}$</p> <p>✓ $x = -21$</p> <p>✓ $PR = PS = 15$ units</p> <p>✓ $MS = MR = 5$ units</p> <p>✓ answer (5)</p>

4.4.2	$\frac{\text{area of } \triangle NPS}{\text{area of quadrilateral PSMR}}$ $\frac{\frac{1}{2} NS.SP}{\frac{1}{2} SP.MS + \frac{1}{2} MR.PR}$ $= \frac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $= \frac{1}{2}$ <p>OR</p> $\triangle NPS \cong \triangle SPM \cong \triangle MPR$ $\frac{\text{area of } \triangle NPS}{\text{area of quadrilateral PSMR}}$ $= \frac{1}{2}$	<p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ congruent</p> <p>✓ answer (2)</p>
[22]		

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QUESTION/VRAAG 3

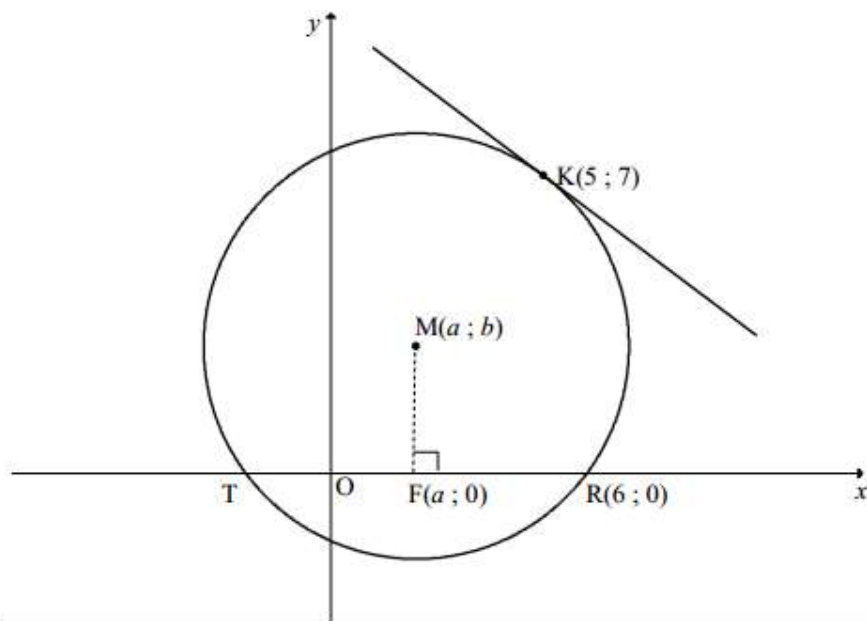


3.1	$m_{AB} = \frac{3 - \frac{1}{2}}{5 - 0}$ $m_{AB} = \frac{1}{2}$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 2/2</div>	<p>✓ substitution</p> <p>✓ answer (2)</p>
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3.2	$m_{CE} = m_{BA} = \frac{1}{2}$ $-4 = \frac{1}{2}(6) + c$ OR/OF $y - (-4) = \frac{1}{2}(x - 6)$ $c = -7$ $y = \frac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; -7) $\frac{x_c + 6}{2} = 0$ $\frac{y_c + (-4)}{2} = -7$ $x_c = -6$ $y_c = -10$ C(-6 ; -10) <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 3/3</div>	✓ D(0 ; -7) ✓ $x_c = -6$ ✓ $y_c = -10$ (3)
3.3.2	Area $\triangle BCD = \frac{1}{2}(7,5)(6)$ $= 22,5$ Area $\triangle ABD = \frac{1}{2}(7,5)(5)$ $= 18,75$ Area ABCD = $22,5 + 18,75 = 41,25 \text{ units}^2$	✓ subst of correct base and height into the area formula ✓ area $\triangle BCD = 22,5$ ✓ area $\triangle ABD = 18,75$ ✓ answer (4)

3.4.1	$K(-6; -4)$	$\checkmark x_K = -6$ $\checkmark y_K = -4$ (2)
3.4.2a	$KC = 6$ units; $KE = 12$ units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter $\triangle KEC = 6 + 12 + \sqrt{180}$ $= 31,42$ units	$\checkmark KC = 6$ units $\checkmark KE = 12$ units $\checkmark CE$ \checkmark answer (4)
3.4.2b	$\tan \hat{KCE} = \frac{KE}{KC} = \frac{12}{6} = 2$ $\hat{KCE} = 63,43^\circ$ OR/OF $\sin \hat{KCE} = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $\hat{KCE} = 63,43^\circ$ OR/OF $m_{CE} = \frac{1}{2}$ $\tan \theta = \frac{1}{2}$ $\theta = 26,57^\circ$ $\hat{KCE} = 90^\circ - 26,57^\circ$ $\hat{KCE} = 63,43^\circ$ OR/OF $KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos \hat{KCE}$ $(12)^2 = (6)^2 + (\sqrt{180})^2 - 2(6)(\sqrt{180})(\cos \hat{KCE})$ $\cos \hat{KCE} = \frac{\sqrt{5}}{5}$ $\hat{KCE} = 63,43^\circ$	\checkmark trig ratio $\checkmark \tan \hat{KCE} = 2$ \checkmark answer (3) \checkmark trig ratio $\checkmark \sin \hat{KCE} = \frac{12}{\sqrt{180}}$ \checkmark answer (3) $\checkmark \tan \theta = \frac{1}{2}$ $\checkmark \theta = 26,57^\circ$ \checkmark answer (3) \checkmark substitution into cosine rule \checkmark trig ratio \checkmark answer (3)

QUESTION/VRAAG 4



4.1.1	$y = x + 1$ $b = a + 1$	$\checkmark b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a - 6)^2 + (b - 0)^2 = (a - 5)^2 + (b - 7)^2$ $(a - 6)^2 + (a + 1)^2 = (a - 5)^2 + (a + 1 - 7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2; 3)$	\checkmark equating radii / solving simultaneously \checkmark substitution $b = a + 1$ $\checkmark 12a = 24$ $\checkmark a = 2$ $\checkmark b = 3$ (5)
4.2.1	$(6 - 2)^2 + (0 - 3)^2 = r^2$ $r = 5$ OR/OF $(2 - 5)^2 + (3 - 7)^2 = r^2$ $r = 5$	\checkmark substitution R and M $\checkmark r = 5$ (2) \checkmark substitution K and M $\checkmark r = 5$ (2)

Answer only 2/2

4.2.2	<p>T(-2 ; 0) TR = 8 units [line from centre \perp to chord]</p> <p>OR/OF</p> <p>M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre \perp to chord]</p> <p>OR/OF</p> <p>$(x-2)^2 + (0-3)^2 = 25$ $x^2 - 4x + 4 + 9 = 25$ $x^2 - 4x - 12 = 0$ $(x-6)(x+2) = 0$ $x = 6$ or $x = -2$ TR = 8 units</p>	<p>✓ T(-2 ; 0) ✓ answer (2)</p> <p>✓ 4 units ✓ answer (2)</p> <p>✓ x values ✓ answer (2)</p>
4.3	<p>$m_{\text{radius}} = \frac{7-3}{5-2}$ $m_{\text{radius}} = \frac{4}{3}$ $m_{\text{tangent}} = -\frac{3}{4}$</p> <p>$7 = -\frac{3}{4}(5) + c$ OR/OF $y - 7 = -\frac{3}{4}(x - 5)$ $c = \frac{43}{4}$ $y = -\frac{3}{4}x + \frac{43}{4}$</p>	<p>✓ substitution ✓ $m_{\text{radius}} = \frac{4}{3}$ ✓ $m_{\text{tangent}} = -\frac{3}{4}$ ✓ substitution ✓ answer (5)</p>
4.4.1	N(2 ; -2)	<p>✓ $x_N = 2$ ✓ $y_N = -2$ (2)</p>
4.4.2	<p>$(-2-2)^2 + (0+2)^2 = r^2$ $r^2 = 20$ $(x-2)^2 + (y+2)^2 = 20$</p>	<p>✓ substitution ✓ $r^2 = 20$ ✓ answer (3)</p>
		[20]

Trigonometrie

Mei/Junie 2024

QUESTION/VRAAG 5

5.1.1	$\sin 220^\circ$ $= -\sin 40^\circ$ $= -p$	$\checkmark -\sin 40^\circ$ \checkmark answer (2)
5.1.2	$\cos^2 50^\circ$ $= \sin^2 40^\circ$ $= p^2$	$\checkmark \sin^2 40$ \checkmark answer (2)
5.1.3	$\cos(-80^\circ)$ $= \cos 80^\circ$ $= 1 - 2\sin^2 40^\circ$ $= 1 - 2p^2$ OR $\cos(-80^\circ)$ $= \cos 80^\circ$ $= \cos(30^\circ + 50^\circ)$ $= \cos 30^\circ \cos 50^\circ - \sin 30^\circ \sin 50^\circ$ $= \frac{\sqrt{3}p}{2} - \frac{\sqrt{1-p^2}}{2}$	$\checkmark \cos 80^\circ$ \checkmark double angle \checkmark answer (3) $\checkmark \cos 80^\circ$ \checkmark expansion \checkmark answer (3)
5.2.1	$\text{LHS} = \tan x(1 - \cos^2 x) + \cos^2 x$ $= \frac{\sin x}{\cos x}(\sin^2 x) + \cos^2 x$ $= \frac{\sin^3 x + \cos^3 x}{\cos x}$ $= \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cos x}$ $= \frac{(\sin x + \cos x)(1 - \sin x \cos x)}{\cos x}$ $= \text{RHS}$ OR	$\checkmark \frac{\sin x}{\cos x}$ $\checkmark \sin^2 x$ \checkmark simplification \checkmark factorisation of cubes $\checkmark \sin^2 x + \cos^2 x = 1$ (5)

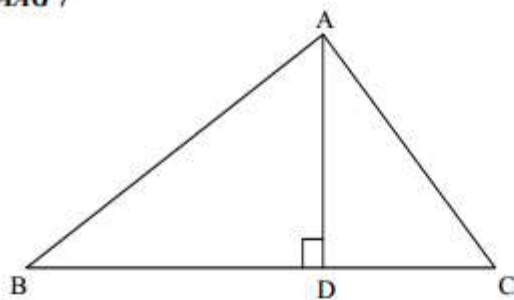
	$\begin{aligned} \text{RHS} &= \frac{(\sin x + \cos x)(1 - \sin x \cos x)}{\cos x} \\ &= \frac{\sin x - \sin^2 x \cos x + \cos x - \sin x \cos^2 x}{\cos x} \\ &= \tan x - \sin^2 x + 1 - \sin x \cos x \\ &= \tan x + \cos^2 x - \sin x \cos x \\ &= \tan x \left(1 - \frac{\sin x \cos x}{\tan x} \right) + \cos^2 x \\ &= \tan x \left(1 - \frac{\sin x \cos x}{\frac{\sin x}{\cos x}} \right) + \cos^2 x \\ &= \tan x (1 - \cos^2 x) + \cos^2 x \\ &= \text{LHS} \end{aligned}$	<p>✓ multiplication</p> <p>✓ ÷ by $\cos x$</p> <p>✓ $-\sin^2 x + 1 = \cos^2 x$</p> <p>✓ factorisation</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>(5)</p>
5.2.2	$\cos x = 0$ or where $\tan x$ is undefined $x = 90^\circ + k \cdot 360^\circ$ or $x = 270^\circ + k \cdot 360^\circ$ $x = 90^\circ$ or $x = -90^\circ$	<p>✓ $\cos x = 0$ or $\tan x$ undefined</p> <p>✓ $x = 90^\circ$ ✓ $x = -90^\circ$</p> <p>(3)</p>
5.3.1	$\begin{aligned} &\frac{\sin 150^\circ + \cos^2 x - 1}{2} \\ &= \frac{\sin 30^\circ + \cos^2 x - 1}{2} \\ &= \frac{\frac{1}{2} - (1 - \cos^2 x)}{2} \\ &= \left(\frac{1}{2} - \sin^2 x \right) \times \frac{1}{2} \\ &= \frac{1 - 2\sin^2 x}{4} \\ &= \frac{\cos 2x}{4} \end{aligned}$	<p>✓ $\sin 30^\circ$</p> <p>✓ $\sin 30^\circ = \frac{1}{2}$ ✓ factor</p> <p>✓ $1 - \cos^2 x = \sin^2 x$</p> <p>✓ simplification</p> <p>✓ answer in terms of $\cos 2x$</p> <p>(6)</p>
5.3.2	$\begin{aligned} \frac{\sin 150^\circ + \cos^2 x - 1}{2} &= \frac{1}{25} \\ \frac{\cos 2x}{4} &= \frac{1}{25} \\ \cos 2x &= \frac{4}{25} \\ \text{ref} \angle &= 80,79...^\circ \\ 2x &= 80,79...^\circ + k \cdot 360^\circ \quad \text{or} \quad 2x = 279,20...^\circ + k \cdot 360^\circ \\ x &= 40,40^\circ + k \cdot 180^\circ \quad \text{or} \quad x = 139,60^\circ + k \cdot 180^\circ ; k \in \mathbb{Z} \end{aligned}$	<p>✓ answer 5.3.1 = $\frac{1}{25}$</p> <p>✓ $2x = 80,79^\circ$</p> <p>✓ $2x = 279,20...^\circ$</p> <p>✓ $x = 40,40^\circ$ and $x = 139,60^\circ$</p> <p>✓ $+ k \cdot 180^\circ ; k \in \mathbb{Z}$</p> <p>(5)</p>

	<p>OR</p> $\frac{\sin 150^\circ + \cos^2 x - 1}{2} = \frac{1}{25}$ $\sin 150^\circ + \cos^2 x - 1 = \frac{2}{25}$ $\sin 30^\circ + \cos^2 x - 1 = \frac{2}{25}$ $\cos^2 x = \frac{29}{50}$ $\cos x = \pm \sqrt{\frac{29}{50}}$ <p> $x = 40,40^\circ + k.360^\circ$ or $x = 319,60^\circ + k.360^\circ ; k \in \mathbb{Z}$ or $x = 139,60^\circ + k.360^\circ$ or $x = 220,40^\circ + k.360^\circ ; k \in \mathbb{Z}$ </p>	$\checkmark \cos^2 x = \frac{29}{50}$ $\checkmark x = 40,40^\circ \quad \checkmark x = 139,60^\circ$ $\checkmark x = 220,40^\circ \text{ and } x = 319,60^\circ$ $\checkmark + k.360^\circ ; \quad k \in \mathbb{Z}$ <p style="text-align: right;">(5)</p>
		[26]

QUESTION/VRAAG 6

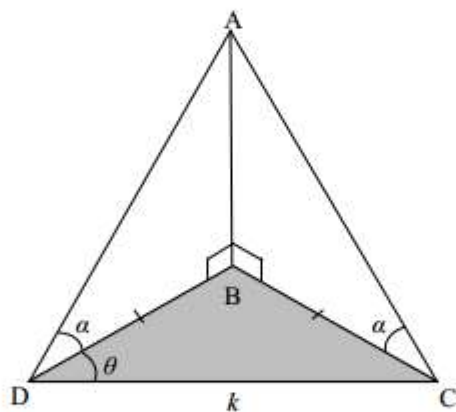
6.1	Period = 360°	✓ 360° (1)
6.2	Amplitude = 1	✓ 1 (1)
6.3	$a = -45^\circ$	✓ $a = -45^\circ$ (1)
6.4	$\sin 2x = k$ $k = \sin(2 \times 165^\circ)$ OR $k = \sin(2 \times (-75^\circ))$ $k = \sin 330^\circ$ $k = \sin(-150^\circ)$ $k = -\sin 30^\circ$ $k = -\frac{1}{2}$ OR $k = \cos(165^\circ - 45^\circ)$ OR $k = \cos(-75^\circ - 45^\circ)$ $k = \cos 120^\circ$ $k = \cos(-120^\circ)$ $k = -\cos 60^\circ$ $k = -\frac{1}{2}$	✓ $-\sin 30^\circ$ ✓ $-\frac{1}{2}$ (2) ✓ $-\cos 60^\circ$ ✓ $-\frac{1}{2}$ (2)
6.5	Points of intersection are translated 60° to the left $x = -15^\circ$	✓ $x = -15^\circ$ (1)
6.6	$\sqrt{2} \sin 2x = \sin x + \cos x$ $\sin 2x = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$ $\sin 2x = \sin 45^\circ \sin x + \cos 45^\circ \cos x$ $\sin 2x = \cos(45^\circ - x)$ OR $\sin 2x = \cos(x - 45^\circ)$ \therefore 2 roots in the interval $x \in [-90^\circ; 90^\circ]$	✓ division by $\sqrt{2}$ ✓ special angles ✓ $\cos(45^\circ - x)$ or $\cos(x - 45^\circ)$ ✓ answer (4)
		[10]

7.1



7.1.1	$\sin \hat{B} = \frac{AD}{AB}$ $AD = AB \sin \hat{B}$	$\checkmark \sin \hat{B} = \frac{AD}{AB}$ $\checkmark \text{ answer}$
7.1.2	$\text{Area of } \triangle ABC = \frac{1}{2}(BC)(AD)$ $\therefore \text{Area of } \triangle ABC = \frac{1}{2}(BC)(AB) \sin \hat{B}$	$\checkmark \frac{1}{2}(BC)(AD)$

7.2



7.2.1

In $\triangle ADB$

$$\sin \alpha = \frac{AB}{AD}$$

$$AD = \frac{AB}{\sin \alpha}$$

In $\triangle ABC$

$$\sin \alpha = \frac{AB}{AC}$$

$$AC = \frac{AB}{\sin \alpha}$$

$AD = AC$

OR

In $\triangle ADB$ and $\triangle ACB$

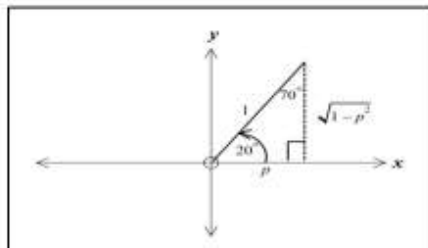
	$AB = AB$ [common side] $\hat{A}BD = \hat{A}BC = 90^\circ$ [given] $BD = BC$ [given] $\triangle ADB \equiv \triangle ACB$ [S \angle S] $\therefore AD = AC$	$\checkmark \triangle ADB \equiv \triangle ACB \checkmark R$ (2)
	OR In $\triangle ADB$ and $\triangle ACB$ $\hat{A}DB = \hat{A}CB = \alpha$ [given] $\hat{A}BD = \hat{A}BC = 90^\circ$ [given] $AB = AB$ OR $BD = BC$ [common side OR given] $\therefore \triangle ADB \equiv \triangle ACB$ [$\angle\angle$ S] $\therefore AD = AC$	$\checkmark \triangle ADB \equiv \triangle ACB \checkmark R$ (2)
	OR $AD^2 = AB^2 + DB^2$ [Pythagoras] $AC^2 = AB^2 + BC^2$ [Pythagoras] But $DB = BC$ [given] $\therefore AD^2 = AC^2$ $\therefore AD = AC$	\checkmark both Pythagoras statements $\checkmark DB = BC$ (2)
7.2.2	$\frac{BD}{\sin \theta} = \frac{k}{\sin(180^\circ - 2\theta)}$ $BD = \frac{k \sin \theta}{\sin 2\theta}$ $BD = \frac{k \sin \theta}{2 \sin \theta \cos \theta}$ $BD = \frac{k}{2 \cos \theta}$ OR $BC^2 = k^2 + BD^2 - 2k(BD)\cos \theta$ $BD^2 = k^2 + BD^2 - 2k(BD)\cos \theta$ $k^2 - 2k(BD)\cos \theta = 0$ $2k(BD)\cos \theta = k^2$ $\therefore BD = \frac{k}{2 \cos \theta}$	\checkmark substitution of $(180^\circ - 2\theta)$ into sine rule \checkmark reduction \checkmark double angle (3) \checkmark substitution into cosine-rule \checkmark substitution BC with BD into cosine-rule \checkmark simplification in terms of BD (3)

7.2.3	$\text{Area of } \triangle BCD = \frac{1}{2}(\overline{DC})(\overline{BD})(\sin \hat{CDB})$ $= \frac{1}{2}k \left(\frac{k}{2 \cos \theta} \right) \sin \theta$ $= \frac{1}{4}k^2 \tan \theta$ <p>OR</p> $\text{Area of } \triangle BCD = \frac{1}{2}(\overline{BD})(\overline{BC})(\sin(180^\circ - 2\theta))$ $= \frac{1}{2} \left(\frac{k}{2 \cos \theta} \right) \left(\frac{k}{2 \cos \theta} \right) (\sin 2\theta)$ $= \frac{2k^2 \sin \theta \cos \theta}{8 \cos \theta \cos \theta}$ $= \frac{1}{4}k^2 \tan \theta$	✓ substitution into area rule ✓ $\frac{\sin \theta}{\cos \theta} = \tan \theta$ ✓ $\frac{1}{4}k^2 \tan \theta$ (3)
		✓ substitution into area rule ✓ $\frac{\sin \theta}{\cos \theta} = \tan \theta$ ✓ $\frac{1}{4}k^2 \tan \theta$ (3)
		[11]

Mei/Junie 2023

QUESTION/VRAAG 5

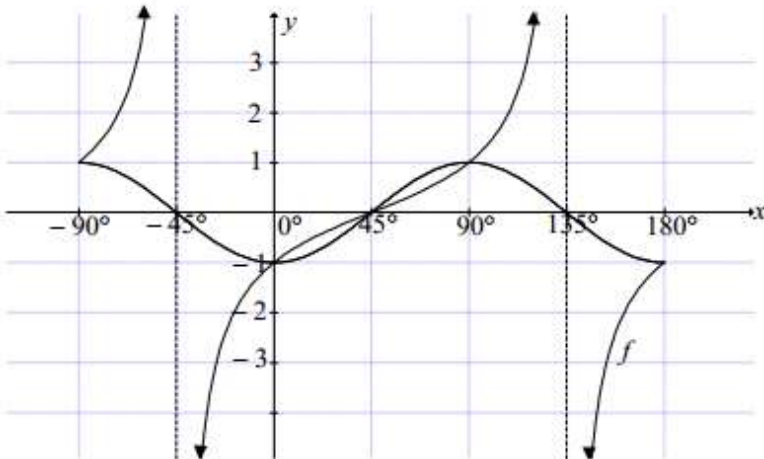
5.1	$\frac{1 - \sin(-\theta)\cos(90^\circ + \theta)}{\cos(\theta - 360^\circ)}$ $= \frac{1 - (-\sin \theta)(-\sin \theta)}{\cos \theta}$ $= \frac{1 - \sin^2 \theta}{\cos \theta}$ $= \frac{\cos^2 \theta}{\cos \theta}$ $= \cos \theta$	✓ $-\sin \theta$ ✓ $-\sin \theta$ ✓ $\cos \theta$ ✓ $\cos^2 \theta$ ✓ answer (5)
5.2.1	$\cos 200^\circ$ $= -\cos 20^\circ$ $= -p$	✓ reduction ✓ answer (2)
5.2.2	$\sin(-70^\circ)$ $= -\sin 70^\circ$ $= -\cos 20^\circ$ $= -p$ <p>OR/OF</p> $\sin(-70^\circ)$ $= -\sin 70^\circ$ $= -p$	✓ reduction ✓ answer ✓ reduction ✓ answer (2)



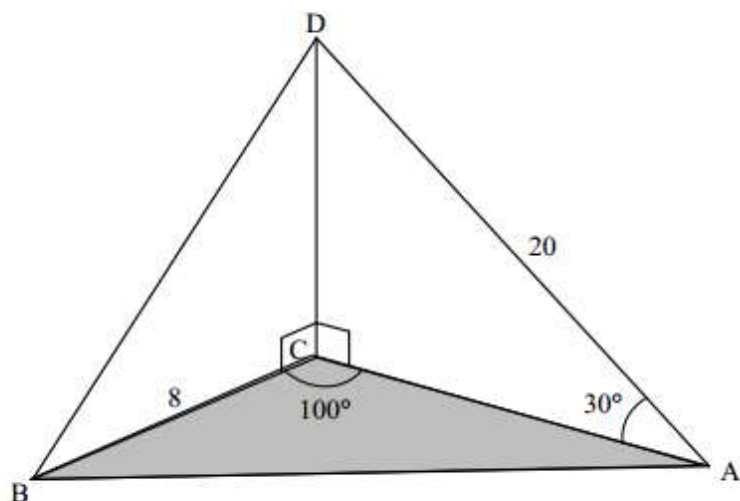
5.2.3	$\sin 10^\circ$ $\cos(2(10^\circ)) = 1 - 2\sin^2 10^\circ$ $2\sin^2 10^\circ = 1 - \cos 20^\circ$ $\sin 10^\circ = \sqrt{\frac{1 - \cos 20^\circ}{2}}$ $\sin 10^\circ = \sqrt{\frac{1 - p}{2}}$ OR/OF $\sin 10^\circ$ $\sin(30^\circ - 20^\circ)$ $= \sin 30^\circ \cos 20^\circ - \cos 30^\circ \sin 20^\circ$ $= \frac{1}{2}p - \frac{\sqrt{3}}{2}\sqrt{1 - p^2} = \frac{p - \sqrt{3}\sqrt{1 - p^2}}{2}$ OR/OF	✓ double angle ✓ $\sin 10^\circ$ as subject ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)
	$\sin 10^\circ$ $\sin(70^\circ - 60^\circ)$ $= \sin 70^\circ \cos 60^\circ - \cos 70^\circ \sin 60^\circ$ $= p \cdot \frac{1}{2} - \sqrt{1 - p^2} \times \frac{\sqrt{3}}{2} = \frac{p - \sqrt{3}\sqrt{1 - p^2}}{2}$ OR/OF $\sin 10^\circ$ $= \cos 80^\circ$ $\cos(60^\circ + 20^\circ)$ $= \cos 60^\circ \cos 20^\circ - \sin 60^\circ \sin 20^\circ$ $= \frac{1}{2}p - \frac{\sqrt{3}}{2}\sqrt{1 - p^2}$	✓ using special angle ✓ expanding ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)

5.3	$\cos(A + 55^\circ)\cos(A + 10^\circ) + \sin(A + 55^\circ)\sin(A + 10^\circ)$ $= \cos[A + 55^\circ - (A + 10^\circ)]$ $= \cos 45^\circ$ $= \frac{1}{\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}}{2}$	✓✓ compound identity ✓ answer (3)
5.4.1	$\text{LHS} = \frac{\cos 2x + \sin 2x - \cos^2 x}{\sin x - 2 \cos x} \qquad \text{RHS} = -\sin x$ $= \frac{\cos^2 x - \sin^2 x + 2 \sin x \cos x - \cos^2 x}{\sin x - 2 \cos x}$ $= \frac{-\sin^2 x + 2 \sin x \cos x}{\sin x - 2 \cos x}$ $= \frac{-\sin x(\sin x - 2 \cos x)}{\sin x - 2 \cos x}$ $= -\sin x$ $\therefore \text{LHS} = \text{RHS}$	✓ $\cos^2 x - \sin^2 x$ ✓ $2 \sin x \cos x$ ✓ common factor of $-\sin x$ (3)
5.4.2	$\frac{\cos 2x + \sin 2x - \cos^2 x}{-3 \sin^2 x + 6 \sin x \cos x}$ $= \frac{\cos 2x + \sin 2x - \cos^2 x}{-3 \sin x(\sin x - 2 \cos x)}$ $= \frac{\cos 2x + \sin 2x - \cos^2 x}{(\sin x - 2 \cos x)} \times \frac{1}{-3 \sin x}$ $= (-\sin x) \times \frac{1}{-3 \sin x}$ $= \frac{1}{3}$	✓ common factor of $-3 \sin x$ ✓ substitution ✓ answer (3)
5.5.1	$3 \tan 4x = -2 \cos 4x$ $3 \left(\frac{\sin 4x}{\cos 4x} \right) = -2 \cos 4x$ $3 \sin 4x + 2 \cos^2 4x = 0$ $3 \sin 4x + 2(1 - \sin^2 4x) = 0$ $-2 \sin^2 4x + 3 \sin 4x + 2 = 0$ $2 \sin^2 4x - 3 \sin 4x - 2 = 0$ $(2 \sin 4x + 1)(\sin 4x - 2) = 0$ $\sin 4x = -\frac{1}{2} \quad \text{or} \quad \sin 4x \neq 2$	✓ identity ✓ $1 - \sin^2 4x$ ✓ standard form ✓ factors (4)
5.5.2	$\sin 4x = -\frac{1}{2}$ $\text{ref. } \angle = 30^\circ$ $4x = 210^\circ + k \cdot 360^\circ \qquad \text{or} \qquad 4x = 330^\circ + k \cdot 360^\circ$ $x = 52,5^\circ + k \cdot 90^\circ ; k \in \mathbb{Z} \qquad \qquad x = 82,5^\circ + k \cdot 90^\circ ; k \in \mathbb{Z}$	✓ $210^\circ ; 330^\circ$ ✓ $52,5^\circ ; 82,5^\circ$ ✓ $k \cdot 90^\circ ; k \in \mathbb{Z}$ (3)
		[28]

QUESTION/VRAAG 6

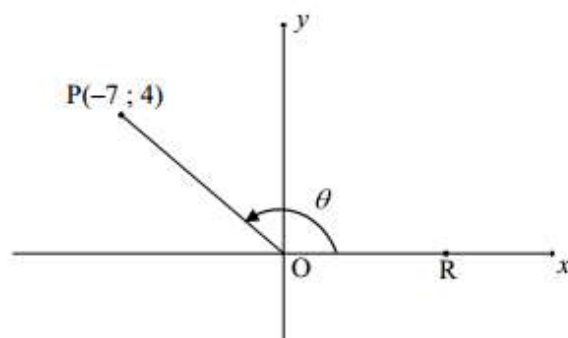
6.1	Period = 180°	✓ answer (1)
6.2		✓ x-intercepts ✓ turning points ✓ end points (3)
6.3	$y \in [-1; 1]$ OR/OF $-1 \leq y \leq 1$	✓ answer (1)
6.4	$g(x) = -\cos 2x$ $g(x + 45^\circ) = -\cos 2(x + 45^\circ)$ $= -\cos(2x + 90^\circ)$ $= \sin 2x$	✓ $-\cos 2(x + 45^\circ)$ ✓ answer (2)
6.5.1	$x \in (-90^\circ; -45^\circ)$ OR/OF $-90^\circ < x < -45^\circ$	✓✓ $x \in (-90^\circ; -45^\circ)$ (2)
6.5.2	$2 \cos 2x - 1 > 0$ $\cos 2x > \frac{1}{2}$ $-\cos 2x < -\frac{1}{2}$ $x \in (-30^\circ; 30^\circ)$ OR/OF $-30^\circ < x < 30^\circ$	✓ $\cos 2x > \frac{1}{2}$ ✓ $-\cos 2x < -\frac{1}{2}$ ✓ $x = \pm 30^\circ$ ✓ interval (4)
		[13]

QUESTION/VRAAG 7



7.1.1	$\frac{AC}{20} = \cos 30^\circ$ $AC = 20 \cos 30^\circ$ $AC = 10\sqrt{3} = 17,32 \text{ units}$ <p>OR/OF</p> $\frac{AC}{\sin 60^\circ} = \frac{20}{\sin 90^\circ}$ $\therefore AC = 20 \sin 60 = 17,32$	✓ trig ratio ✓ answer ✓ trig ratio ✓ answer	(2) (2)
7.1.2	$AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cos \hat{C}$ $AB^2 = (10\sqrt{3})^2 + 8^2 - 2(10\sqrt{3})(8) \cos 100^\circ$ $AB = 20,30 \text{ units}$	✓ cosine formula ✓ substitution into cosine formula ✓ answer	(3)
7.2	$\frac{\sin \hat{A}DB}{AB} = \frac{\sin \hat{A}BD}{AD}$ $\frac{\sin \hat{A}DB}{20,3} = \frac{\sin 73,4^\circ}{20}$ $\sin \hat{A}DB = \frac{20,3 \sin 73,4^\circ}{20}$ $\hat{A}DB = 76,58^\circ$	✓ sine formula in $\triangle ABD$ ✓ substitution into sine formula ✓ answer	(3)

QUESTION/VRAAG 5



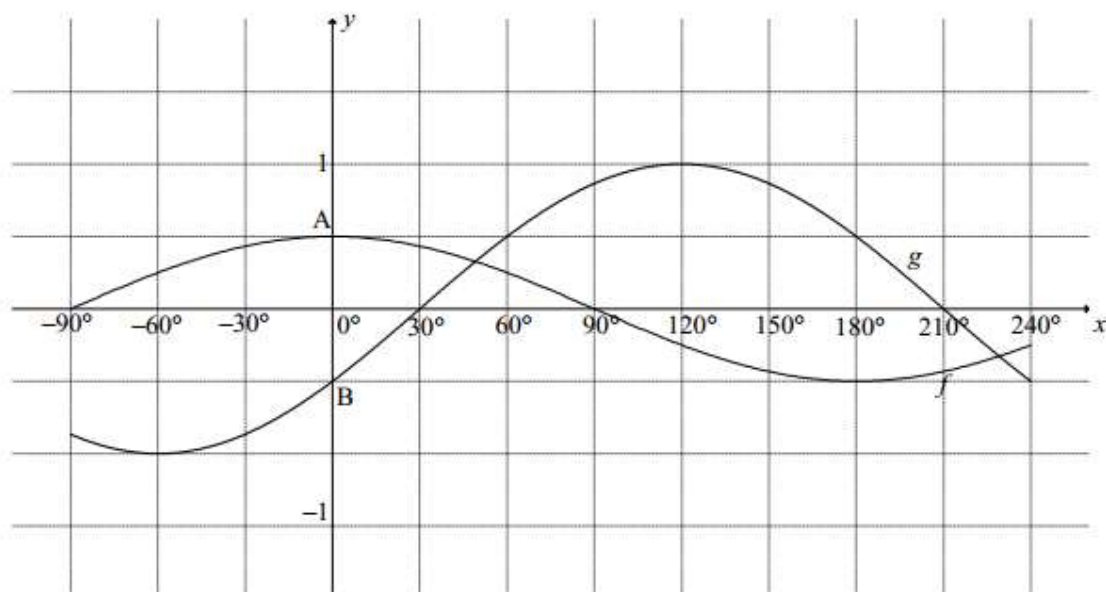
5.1.1	$OP = \sqrt{(-7)^2 + (4)^2}$ $= \sqrt{65}$	✓ substitution ✓ answer (2)
5.1.2(a)	$\tan \theta = \frac{4}{-7}$	✓ answer (1)
5.1.2(b)	$\cos(\theta - 180^\circ) = -\cos \theta$ $= \frac{7}{\sqrt{65}}$	✓ reduction ✓ answer (2)
5.2	$\sin x \cos x + \sin x = 3 \cos^2 x + 3 \cos x$ $\sin x \cos x + \sin x - 3 \cos^2 x - 3 \cos x = 0$ $\sin x(\cos x + 1) - 3 \cos x(\cos x + 1) = 0$ $(\cos x + 1)(\sin x - 3 \cos x) = 0$ $\cos x = -1 \quad \text{or} \quad \sin x = 3 \cos x$ $\tan x = 3$ $x = 180^\circ + k.360^\circ \quad \text{or} \quad x = 71,57^\circ + k.180^\circ ; k \in Z$ <p>OR/OF</p> $\sin x \cos x + \sin x = 3 \cos^2 x + 3 \cos x$ $\sin x \cos x + \sin x - 3 \cos^2 x - 3 \cos x = 0$ $\sin x(\cos x + 1) - 3 \cos x(\cos x + 1) = 0$ $(\cos x + 1)(\sin x - 3 \cos x) = 0$ $\cos x = -1 \quad \text{or} \quad \sin x = 3 \cos x$ $\tan x = 3$ $x = 180^\circ + k.360^\circ \quad \text{or} \quad x = 71,57^\circ + k.360^\circ \quad \text{or}$ $x = 251,57^\circ + k.360^\circ ; k \in Z$	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x = 180^\circ$ ✓ $x = 71,57^\circ$ ✓ $+ k.180^\circ ; k \in Z$ (7)
	$\sin x \cos x + \sin x = 3 \cos^2 x + 3 \cos x$ $\sin x \cos x + \sin x - 3 \cos^2 x - 3 \cos x = 0$ $\sin x(\cos x + 1) - 3 \cos x(\cos x + 1) = 0$ $(\cos x + 1)(\sin x - 3 \cos x) = 0$ $\cos x = -1 \quad \text{or} \quad \sin x = 3 \cos x$ $\tan x = 3$ $x = 180^\circ + k.360^\circ \quad \text{or} \quad x = 71,57^\circ + k.360^\circ \quad \text{or}$ $x = 251,57^\circ + k.360^\circ ; k \in Z$	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x = 180^\circ$ ✓ $x = 71,57^\circ$ and $251,57^\circ$ ✓ $+ k.360^\circ ; k \in Z$ (7)

5.3.1	$\begin{aligned} \text{LHS} &= \frac{\sin 3x}{1 - \cos 3x} \times \frac{1 + \cos 3x}{1 + \cos 3x} \\ &= \frac{(\sin 3x)(1 + \cos 3x)}{(1 - \cos 3x)(1 + \cos 3x)} \\ &= \frac{(\sin 3x)(1 + \cos 3x)}{1 - \cos^2 3x} \\ &= \frac{(\sin 3x)(1 + \cos 3x)}{\sin^2 3x} \\ &= \frac{1 + \cos 3x}{\sin 3x} \\ &= \text{RHS} \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \text{LHS} &= \frac{\sin 3x}{1 - \cos 3x} \times \frac{\sin 3x}{\sin 3x} \\ &= \frac{\sin^2 3x}{\sin 3x(1 - \cos 3x)} \\ &= \frac{1 - \cos^2 3x}{\sin 3x(1 - \cos 3x)} \\ &= \frac{(1 - \cos 3x)(1 + \cos 3x)}{\sin 3x(1 - \cos 3x)} \\ &= \frac{1 + \cos 3x}{\sin 3x} \\ &= \text{RHS} \end{aligned}$	<p>✓ multiply by “1”</p> <p>✓ $1 - \cos^2 3x$</p> <p>✓ square identity</p> <p>(3)</p> <p>✓ multiply by “1”</p> <p>✓ square identity</p> <p>✓ factors</p> <p>(3)</p>
5.3.2	undefined when $\sin 3x = 0$ and $1 - \cos 3x = 0$ $3x = 0^\circ$ or $3x = 180^\circ$ and $3x = 0^\circ$ or $3x = 360^\circ$ $x = 0^\circ$ or $x = 60^\circ$	<p>✓ $\sin 3x = 0$ and $1 - \cos 3x = 0$</p> <p>✓ 0° ✓ 60°</p> <p>(3)</p>
[18]		

QUESTION/VRAAG 6

6.1	$\frac{\sin 10^\circ}{\cos 440^\circ} + \tan(360^\circ - \theta) \cdot \sin 2\theta$ $= \frac{\cos 80^\circ}{\cos 80^\circ} - \tan \theta (2 \sin \theta \cos \theta)$ $= 1 - \frac{\sin \theta}{\cos \theta} (2 \sin \theta \cos \theta)$ $= 1 - 2 \sin^2 \theta$ $= \cos 2\theta$	<p>✓ $-\tan \theta$ ✓ $\cos 80^\circ$ ✓ co-ratio ✓ double angle</p> <p>✓ quotient identity</p> <p>✓ answer</p> <p>(6)</p>
6.2.1	$\sin(60^\circ + 2x) + \sin(60^\circ - 2x) = k \cos 2x$ $(\sin 60^\circ \cos 2x + \cos 60^\circ \sin 2x) + (\sin 60^\circ \cos 2x - \cos 60^\circ \sin 2x) = k \cos 2x$ $2 \sin 60^\circ \cos 2x = k \cos 2x$ $2 \left(\frac{\sqrt{3}}{2} \right) \cos 2x = k \cos 2x$ $\therefore k = \sqrt{3}$	<p>✓ both expansions correct</p> <p>✓ special \angles</p> <p>✓ answer</p> <p>(3)</p>
6.2.2	$\tan 60^\circ [\sin(60^\circ + 2x) + \sin(60^\circ - 2x)]$ $= \tan 60^\circ [k \cos 2x]$ $= \sqrt{3} (\sqrt{3} \cos 2x)$ $= 3(2 \cos^2 x - 1)$ $= 3(2(\sqrt{t})^2 - 1)$ $= 6(\sqrt{t})^2 - 3$ $= 6t - 3$	<p>✓ special \angle</p> <p>✓ double \angles</p> <p>✓ answer i.t.o t</p> <p>(3)</p>
[12]		

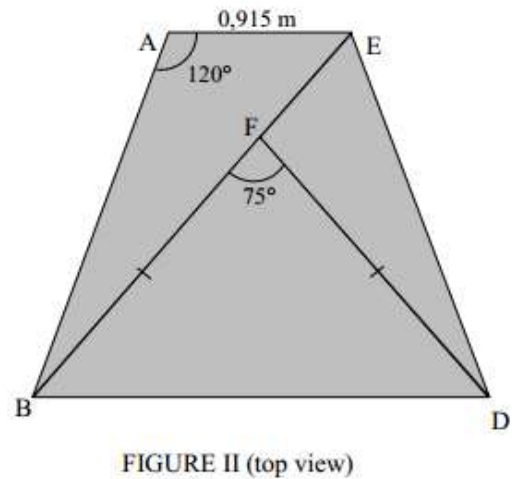
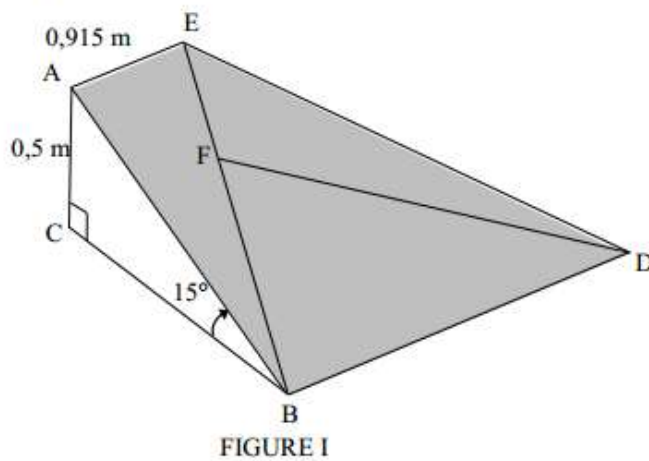
QUESTION/VRAAG 7



7.1	$A\left(0; \frac{1}{2}\right) \quad B\left(0; -\frac{1}{2}\right)$ $AB = \frac{1}{2} - \left(-\frac{1}{2}\right)$ $= 1 \text{ unit}$	✓ y-values ✓ answer Answer only 2/2	(2)
7.2	Range of $f: y \in \left[-\frac{1}{2}; \frac{1}{2}\right]$ Range of $3f(x) + 2: y \in \left[\frac{1}{2}; 3\frac{1}{2}\right]$ OR/OF $\frac{1}{2} \leq y \leq 3\frac{1}{2}$	✓ critical values ✓ answer	(2)
7.3	$x = 90^\circ$	✓✓ $x = 90^\circ$	(2)
7.4.1	$x \in (30^\circ; 90^\circ) \cup (210^\circ; 240^\circ]$ OR/OF $30^\circ < x < 90^\circ$ or $210^\circ < x \leq 240^\circ$	✓ $x \in (30^\circ; 90^\circ)$ ✓ $(210^\circ; 240^\circ]$ ✓ $30^\circ < x < 90^\circ$ ✓ $210^\circ < x \leq 240^\circ$	(2)
7.4.2	$x \in (-55^\circ; 125^\circ)$ OR/OF $-55^\circ < x < 125^\circ$	✓ critical values ✓ answer ✓ critical values ✓ answer	(2)

[10]

QUESTION/VRAAG 8



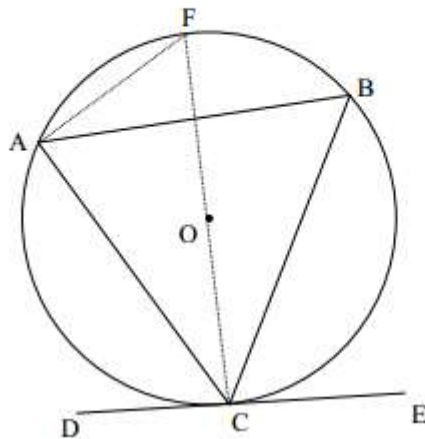
8.1	$\frac{0,5}{AB} = \sin 15^\circ$ $AB = \frac{0,5}{\sin 15^\circ}$ $AB = 1,93 \text{ m}$	✓ trig ratio ✓ answer <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 2/2</div>
8.2	$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos \hat{BAE}$ $BE^2 = (1,93)^2 + (0,915)^2 - 2(1,93)(0,915)(\cos 120^\circ)$ $BE = 2,52 \text{ m}$	✓ correct use of cosine rule ✓ substitution ✓ answer <div style="text-align: right;">(3)</div>
8.3	$BF = FD = \frac{5}{7}(2,52) = 1,80 \text{ m}$ $\text{Area } \triangle BFD = \frac{1}{2}(BF)(FD)\sin \hat{BFD}$ $= \frac{1}{2}(1,8)(1,8)(\sin 75^\circ)$ $= 1,56 \text{ m}^2$	✓ BF ✓ correct substitution into the area rule ✓ answer <div style="text-align: right;">(3)</div>
		[8]

Euklidiese Meetkunde

Mei/Junie 2024

QUESTION/VRAAG 8

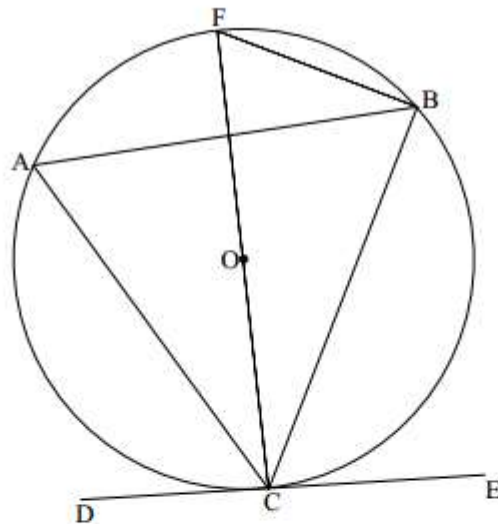
8.1



	Construction: Draw diameter CF and draw AF <i>Konstruksie: Trek middellyn CF en verbind AF</i>	✓ Constr
	$\hat{FCE} = 90^\circ$ [tan \perp radius/raaklyn \perp radius]	✓ S ✓ R
	$\hat{FAC} = 90^\circ$ [\angle in semi circle/ \angle in halwe sirkel]	✓ S/R
	$\hat{FAB} = \hat{FCB}$ [\angle s same segment/ \angle e dieselfde segm]	✓ S/R
	$\therefore \hat{BAC} = \hat{BCE}$	
	$\therefore \hat{BCE} = \hat{A}$	(5)

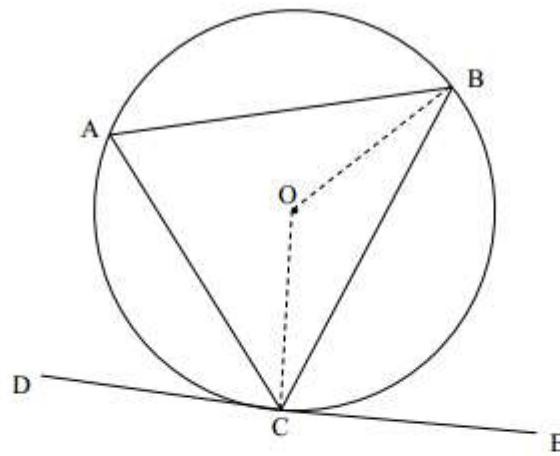
OR

8.1



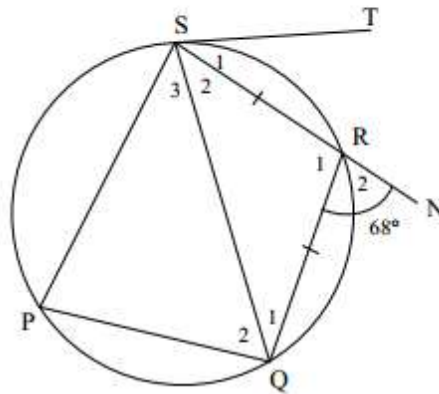
Construction: Draw diameter CF and draw FB <i>Konstruksie: Trek middellyn CF en verbind FB</i>		✓ construction
$\hat{FBC} = 90^\circ$	[\angle in semi circle/ \angle in <i>halwe sirkel</i>]	✓ S / R
$\hat{BFC} + \hat{FCB} = 90^\circ$	[sum of \angle s in Δ / <i>binne \anglee v Δ</i>]	
$\hat{OCE} = 90^\circ$	[tan \perp radius/ <i>raaklyn \perp radius</i>]	✓ S ✓ R
$\therefore \hat{BCE} = \hat{F}$		
but $\hat{A} = \hat{F}$	[\angle s in same seg/ \angle in <i>dies. segment</i>]	✓ S / R
$\therefore \hat{BCE} = \hat{A}$		(5)

OR



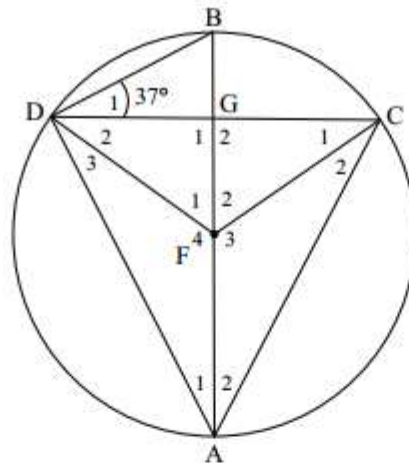
	<p>Construction: Draw radii BO and OC <i>Konstruksie: Trek radiusse BO en OC</i></p> <p>$\hat{OCE} = 90^\circ$ or $\hat{BCE} = 90^\circ - \hat{OCB}$ [tan \perp radius / <i>raaklyn \perp radius</i>]</p> <p>$\hat{OCB} = \hat{OBC}$ [∠s opp equal sides/ <i>∠e teenoor gelyke sye</i>] $\therefore \hat{COB} = 180^\circ - 2\hat{OCB}$ [∠s of Δ/∠e van Δ]</p> <p>$\hat{CAB} = 90^\circ - \hat{OCB}$ [∠ at centre = $2 \times$ ∠ circumf/ <i>midpts ∠ = $2 \times$ omtreks ∠</i>] $\therefore \hat{BCE} = \hat{CAB}$</p>	<p>✓ construction</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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8.2



8.2.1	$\hat{P} = \hat{R}_2 = 68^\circ$	[ext \angle of cyclic quad / buite \angle van kvh]	✓ S ✓ R	(2)
8.2.2	$\hat{Q}_1 = \hat{S}_2$ $\hat{Q}_1 + \hat{S}_2 = 68^\circ$ $\therefore \hat{Q}_1 = 34^\circ$	[\angle s opp equal sides / \angle e teenoor gelyke sye] [ext \angle of Δ / buite \angle van Δ]	✓ S ✓ S	(2)
8.2.3	$\hat{S}_1 = \hat{Q}_1 = 34^\circ$	[tan-chord theorem / \angle tussen rkl en koord]	✓ S ✓ R	(2)
				[11]

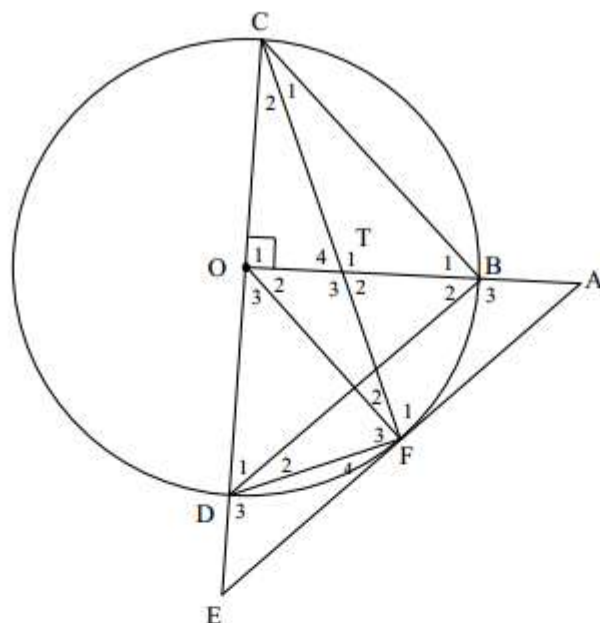
QUESTION/VRAAG 9



9.1	$\hat{A}_2 = \hat{D}_1 = 37^\circ$ $\hat{A}_1 = \hat{A}_2 = 37^\circ$ $\hat{D}_3 = \hat{A}_1 = 37^\circ$ $\hat{C}_2 = \hat{A}_2 = 37^\circ$	<p>[\angles in the same seg/\anglee in dies segment] [BA bisects $\hat{C}\hat{A}\hat{D}$/BA halveer $\hat{C}\hat{A}\hat{D}$] [\angles opp equal sides/\anglee teenoor gelyke sye] [\angles opp equal sides/\anglee teenoor gelyke sye]</p>	<p>✓ S ✓ R ✓✓ any other two statements</p>
9.2	$\hat{A}\hat{D}\hat{G} = 53^\circ$ $\hat{A}_1 = 37^\circ$ $\therefore \hat{G}_1 = 90^\circ$ $\therefore CG = DG$ OR $\hat{F}_2 = 2\hat{D}_1 = 74^\circ$ $\hat{D}_3 = 37^\circ$ $\therefore \hat{D}_2 = 16^\circ$ $\hat{C}_1 = \hat{D}_2 = 16^\circ$ $\therefore \hat{G}_2 = 90^\circ$ $\therefore CG = DG$	<p>[\angle in semi circle/\angle in halwe sirkel] [proved in 9.1/reeds bewys in 9.1] [sum of \angles in Δ/binne \anglee van Δ] [line from centre \perp to chord/ lyn uit midpt. \perp op koord]</p> <p>[\angle at centre = $2 \times \angle$ at circumference/ midpt. \angles = $2 \times$ omtreks \angle] [proved in 9.1/reeds bewys in 9.1] [\angle in semi circle/\angle in halwe sirkel] [\angles opp equal sides/\anglee teenoor gelyke sye] [sum of \angles in Δ/binne \anglee van Δ] [line from centre \perp to chord/ lyn uit midpt. \perp op koord]</p>	<p>✓ S ✓ R ✓ S ✓ R ✓ S ✓ R ✓ S ✓ R</p>

9.3	$\hat{F}_2 = 2\hat{D}_1 = 74^\circ$ OR $\hat{F}_2 = 2\hat{A}_2 = 74^\circ$ [\angle at centre = $2 \times \angle$ at circum./ midpt. $\angle s = 2 \times \text{omtreks } \angle$] $\frac{FG}{20} = \cos 74^\circ$ $FG = 5,51$ $\therefore BG = 14,49$ units OR $\hat{F}_2 = 2\hat{D}_1 = 74^\circ$ [\angle at centre = $2 \times \angle$ at circumference midpt. $\angle = 2 \times \text{omtreks } \angle$] $\frac{FG}{20} = \sin 16^\circ$ $FG = 5,51$ $\therefore BG = 14,49$ units OR $\frac{DG}{20} = \cos 16^\circ$ $DG = 19,23$ $\frac{BG}{19,23} = \tan 37^\circ$ $BG = 14,49$ units OR $\frac{DG}{20} = \cos 16^\circ$ $DG = 19,23$ $FG^2 = FD^2 - DG^2$ [Pythagoras] $FG^2 = 20^2 - (19,23)^2$ $FG = 5,51$ $BG = 20 - 5,51$ $= 14,49$ units	✓ S ✓ trig ratio ✓ FG ✓ answer (4) ✓ S ✓ trig ratio ✓ FG ✓ answer (4) ✓ trig ratio ✓ length of DG ✓ trig ratio ✓ answer (4) ✓ trig ratio ✓ length of DG ✓ correct use of Pythagoras ✓ answer (4)
		[12]

QUESTION/VRAAG 10

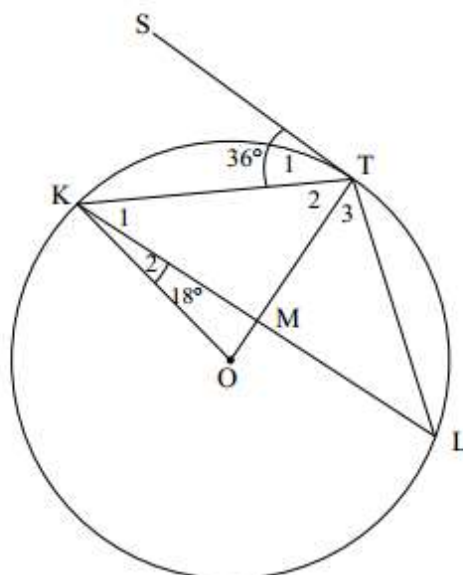


10.1	$\hat{O}_1 = 90^\circ$ $\hat{F}_2 + \hat{F}_3 = 90^\circ$ $\hat{O}_1 = \hat{F}_2 + \hat{F}_3 = 90^\circ$ \therefore TODF is a cyclic quad	[given/gegee] [\angle in semi circle/ \angle in halwe sirkel] [ext \angle = int opp \angle / buite \angle = teenoorst. binne \angle] OR [converse ext \angle of cyclic quad/ omgekeerde buite \angle v kvh]	✓ S ✓ R ✓ S ✓ R	(4)
10.2	$\hat{T}_1 = \hat{T}_3$ But $\hat{D}_3 = \hat{T}_3$ $\therefore \hat{T}_1 = \hat{D}_3$	[vert opp \angle s =/ regoorstaande \angle e] [ext \angle of cyclic quad/ buite \angle v kvh]	✓ S / R ✓ S ✓ R	(3)
10.3	In $\triangle DFE$ and $\triangle TFO$ 1) $\hat{D}_3 = \hat{T}_3$ 2) $\hat{F}_4 = \hat{C}_2$ but $\hat{C}_2 = \hat{F}_2$ $\therefore \hat{F}_4 = \hat{F}_2$ 3) $\hat{E} = \hat{O}_2$ $\triangle TFO \parallel \triangle DFE$	[ext \angle of cyclic quad/ buite \angle v kvh] [tan-chord theorem/ \angle tussen rkl en koord] [\angle s opp equal sides/ \angle e teenoor gelyke sye] [3 rd \angle of \triangle / \angle e van \triangle] [$\angle\angle\angle$]	✓ S ✓ S / R ✓ S ✓ S ✓ S OR R	(5)

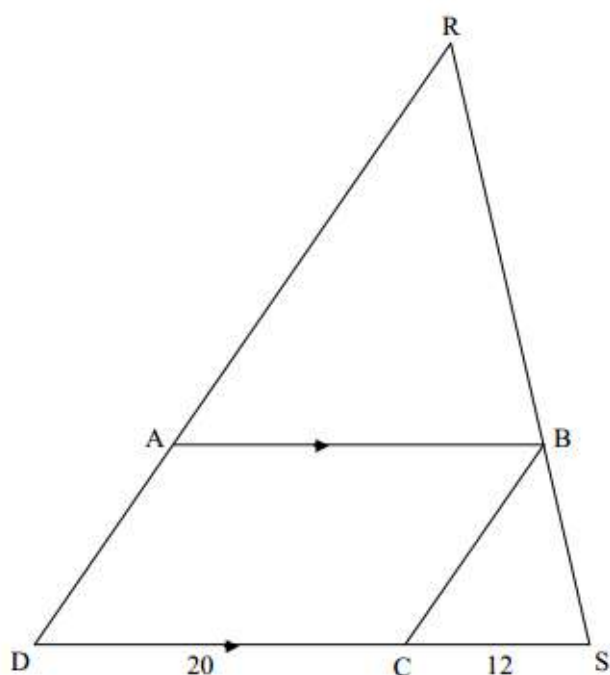
	<p>OR</p> <p>In $\triangle DFE$ and $\triangle TFO$</p> <p>1) $\hat{D}_3 = \hat{T}_3$ [ext \angle of cyclic quad/buite \angle van \triangle] ✓ S</p> <p>2) $\hat{F}_4 = \hat{C}_2$ [tan-chord theorem/\angle tussen rkl en koord] ✓ S / R</p> <p>$\hat{F}_2 + \hat{F}_3 = 90^\circ$ [\angle in semi circle/\angle in halwe sirkel]</p> <p>$\hat{D}_1 + \hat{D}_2 = 90^\circ - \hat{C}_2$ [sum of \angles in \triangle/binne \anglee van \triangle]</p> <p>$\hat{E} = 90^\circ - 2\hat{F}_4$ [ext \angle of \triangle/buite \angle van \triangle] ✓ S</p> <p>$\hat{O}_3 = 2\hat{C}_2$ [\angle at centre = $2 \times \angle$ at circumference/ midpt. \angles = $2 \times$ omtreks \angle]</p> <p>$\hat{O}_2 = 90^\circ - 2\hat{F}_4$ [\angles on a str line/\anglee op 'n reguitlyn] ✓ S</p> <p>$\hat{O}_2 = \hat{E}$</p> <p>3) $\therefore \hat{F}_4 = \hat{F}_2$ [3^{rd} \angle of \triangle/\anglee van \triangle] ✓ S OR R</p> <p>$\triangle TFO \parallel \triangle DFE$ [$\angle \angle \angle$] (5)</p>	
10.4	<p>$\hat{B}_2 = \hat{D}_1$ [\angles opp equal sides/\anglee teenoor gelyke sye] ✓ S / R</p> <p>$\hat{B}_2 = \hat{E}$ [given/gegee]</p> <p>$\therefore \hat{D}_1 = \hat{E}$</p> <p>$\therefore DB \parallel EA$ [corresp \angles =/ooreenkomstige \anglee gelyk] ✓ R</p>	(2)
10.5	<p>In $\triangle OEA$</p> <p>$DB \parallel EA$ [proven/reeds bewys]</p> <p>$\frac{OD}{DE} = \frac{OB}{BA}$ [line \parallel one side of \triangle/lyn \parallel een sy van \triangle] ✓ R</p> <p>OR</p> <p>[prop theorem; $DB \parallel EA$/ eweredigheid stelling; $DB \parallel EA$]</p> <p>$\therefore DE = \frac{DO \cdot AB}{OB}$ ✓ S</p> <p>$\frac{FO}{FE} = \frac{TO}{DE}$ [$\triangle TFO \parallel \triangle DFE$] ✓ S / R</p> <p>$DE = \frac{TO \cdot FE}{FO}$ ✓ S</p> <p>$\therefore \frac{DO \cdot AB}{OB} = \frac{TO \cdot FE}{FO}$ ✓ S</p> <p>$\therefore \frac{DO \cdot AB}{DO} = \frac{TO \cdot FE}{DO}$ [DO = OB = FO]</p> <p>$\therefore DO = \frac{TO \cdot FE}{AB}$</p>	(5)
		[19]

QUESTION/VRAAG 8

8.1



8.1.1(a)	$\hat{T}_2 = 54^\circ$ [tan \perp rad]	✓ S ✓R (2)
8.1.1(b)	$\hat{L} = 36^\circ$ [tan - chord theorem]	✓ S ✓R (2)
8.1.1(c)	$\hat{KOT} = 72^\circ$ [\angle at centre = $2 \times \angle$ at circumference] OR/OF $\hat{OKT} = \hat{T}_2 = 54^\circ$ [s opposite = radii] $\hat{KOT} = 180^\circ - (54^\circ + 54^\circ)$ [sum of int \angle 's of Δ] $= 72^\circ$	✓ S ✓R (2) ✓ S/R ✓ S (2)
8.1.2	$\hat{KMO} = 180^\circ - (18^\circ + 72^\circ)$ $= 90^\circ$ [sum of int \angle 's of Δ] $\therefore KM = ML$ [line from centre \perp to chord] OR/OF $\hat{OKT} = 54^\circ$ [s opposite = radii] $\hat{K}_1 = 54^\circ - 18^\circ = 36^\circ$ $\hat{TMK} = 90^\circ$ [sum of int \angle 's of Δ] $\therefore KM = ML$ [line from centre \perp to chord]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3)

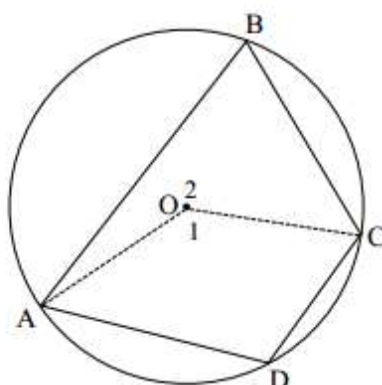


8.2.1	$\frac{DC}{CS} = \frac{20}{12} = \frac{5}{3}$ $\therefore \frac{DC}{CS} = \frac{RB}{BS}$ $\therefore BC \parallel DR$ $\therefore BC \parallel AD$ <p>[converse line \parallel one side of Δ OR sides in the same proportion]</p>	<p>✓ S</p> <p>✓ S</p> <p>✓ R</p> <p>(3)</p>
8.2.2	$\frac{AR}{AD} = \frac{RB}{BS}$ [line \parallel one side of Δ] OR [Prop Theorem $AB \parallel DS$] $\frac{AR}{AD} = \frac{5}{3}$ $\frac{48 - AD}{AD} = \frac{5}{3}$ $\therefore 5AD = 144 - 3AD$ $AD = 18$ $AB = 20$ [opp sides of parm] $\therefore AD : AB = 18 : 20 = 9 : 10$	<p>✓ $\frac{AR}{AD} = \frac{5}{3}$</p> <p>✓ $AD = 18$</p> <p>✓ ratio</p> <p>(3)</p>

	<p>OR/OF</p> $\frac{AR}{RD} = \frac{5}{8} \dots\dots\dots \text{prop thm } AB \parallel DS$ $\frac{AR}{48} = \frac{5}{8}$ $\therefore AR = 30 \text{ and } AD = 18$ $\therefore \frac{AR}{RD} = \frac{AB}{DS} \dots\dots\dots \parallel \Delta's$ $\therefore AB = 20$ $\therefore AB : AD = 18 : 20 = 9 : 10$	<p>✓ $\frac{AR}{RD} = \frac{5}{8}$ ✓ $AD = 18$</p> <p>✓ ratio</p> <p>(3)</p> <p>[15]</p>
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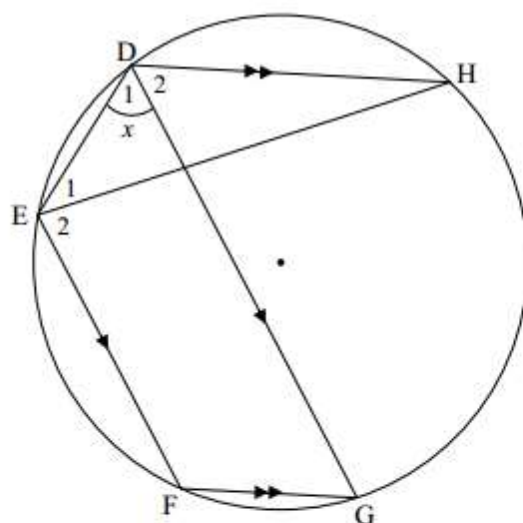
QUESTION/VRAAG 9

9.1



9.1	<p>Constr: Draw radii OA and OC.</p> <p>Proof:</p> $\hat{O}_1 = 2\hat{B} \quad [\angle \text{ at centre} = 2 \times \angle \text{ at circumference}]$ $\hat{O}_2 = 2\hat{D} \quad [\angle \text{ at centre} = 2 \times \angle \text{ at circumference}]$ $\hat{O}_1 + \hat{O}_2 = 360^\circ \quad [\text{revolution}]$ $2\hat{B} + 2\hat{D} = 360^\circ \quad [\text{revolution}]$ $\therefore \hat{B} + \hat{D} = 180^\circ$	<p>✓ Construction</p> <p>✓ S ✓ R</p> <p>✓ S/R</p> <p>✓ S</p> <p>(5)</p>
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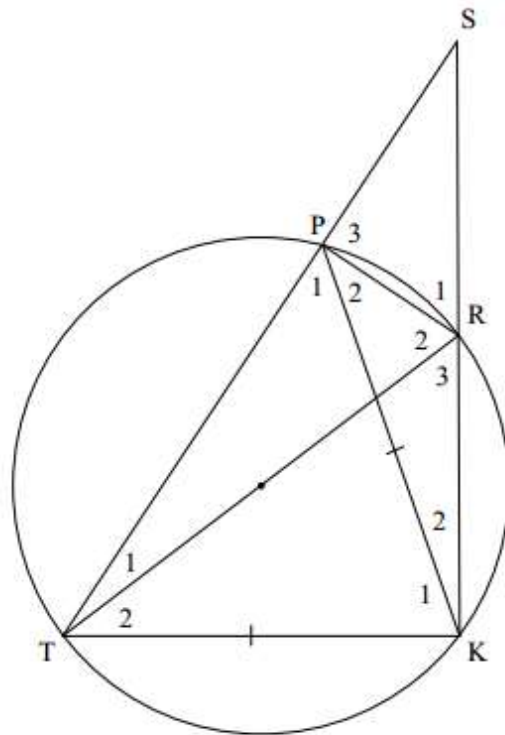
9.2



9.2	$\hat{EFG} = 180^\circ - \hat{D}_1$ [opp \angle 's of cyclic quad] $\therefore \hat{EFG} = 180^\circ - x$ $\hat{EFG} = 180^\circ - \hat{G}$ [co-int \angle 's; $EF \parallel DG$] $\hat{G} = x$ But $\hat{G} = \hat{D}_2$ [alt \angle 's; $DH \parallel FG$] $\therefore \hat{D}_1 = \hat{D}_2 = x$	$\checkmark S \checkmark R$ $\checkmark S / R$ $\checkmark S / R$ <div style="text-align: right;">(4)</div>
		[9]

QUESTION/VRAAG 10

10.1



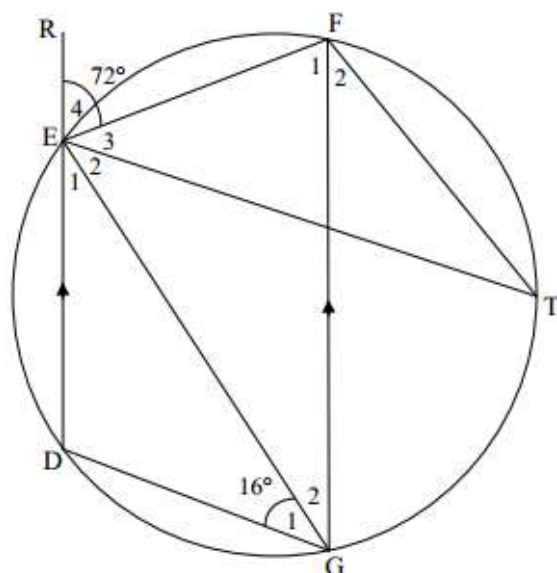
10.1.1	$\hat{T}PR = 90^\circ$ $\hat{S}PR = 90^\circ$ $\therefore SR$ is a diameter	$[\angle \text{ in semi-circle}]$ $[\angle \text{'s on a straight line}]$ $[\text{converse } \angle \text{ in semi-circle}]$	$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark R$	(4)
	<p>OR</p> $\hat{T}KR = 90^\circ$ $\hat{S}PR = 90^\circ$ $\therefore SR$ is a diameter	$[\angle \text{ in semi-circle}]$ $[\text{ext } \angle \text{ of cyclic quad}]$ $[\text{converse } \angle \text{ in semi-circle}]$ <p>OR</p> $[\text{chord subtends a right angle}]$	$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark R$	

10.1.2	$\hat{R}_1 = \hat{P}\hat{T}K$ [ext \angle of cyclic quad] $\hat{P}_1 = \hat{P}\hat{T}K = \hat{R}_1$ [\angle s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext \angle of Δ] $\therefore \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]	$\checkmark S \checkmark R$ $\checkmark S / R$ $\checkmark S \checkmark R$	(5)
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $\Delta SPK \parallel \Delta PRK$ [\angle, \angle, \angle] OR/OF In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of \angle s in Δ] $\Delta SPK \parallel \Delta PRK$	$\checkmark S$ $\checkmark S$ $\checkmark S/R$ $\checkmark S$ $\checkmark S$ $\checkmark S/R$	(3)
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [$\Delta SPK \parallel \Delta PRK$] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] $TK = PK$ [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$	$\checkmark S$ $\checkmark S$ $\checkmark PK^2 = SK.RK$ $\checkmark SK = 2RK$ $\checkmark ST^2 = 6RK^2$	(5)
			[17]

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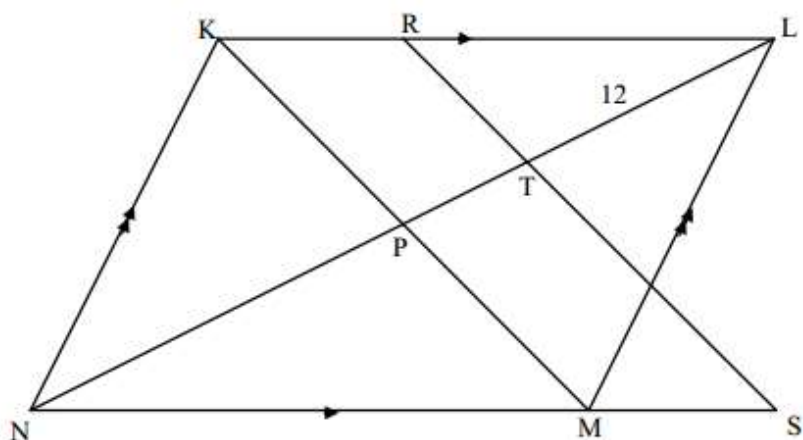
QUESTION/VRAAG 9

9.1



9.1.1	$\widehat{DGF} = \widehat{E}_4 = 72^\circ$ [ext \angle of cyclic quad/ buite \angle v kvh]	✓ S ✓ R (2)
9.1.2	$\widehat{G}_2 = 72^\circ - 16^\circ = 56^\circ$ $\widehat{T} = \widehat{G}_2 = 56^\circ$ [\angle s in the same seg/ \angle e in dies. \odot segment]	✓ S ✓ S / R (2)
9.1.3	$\widehat{F}_1 = \widehat{E}_4 = 72^\circ$ [alt \angle s; DE \parallel GF / verw. \angle e; DE \parallel GF] $\therefore \widehat{GEF} = 52^\circ$ [sum of \angle s in Δ / \angle e van Δ] OR/OF $\widehat{E}_1 = 56^\circ$ [alt \angle s; DE \parallel GF / verw. \angle e; DE \parallel GF] $\therefore \widehat{GEF} = 52^\circ$ [\angle s on a str. line/ \angle e op 'n reguitlyn]	✓ S / R ✓ S (2) ✓ S / R ✓ S (2)

9.2

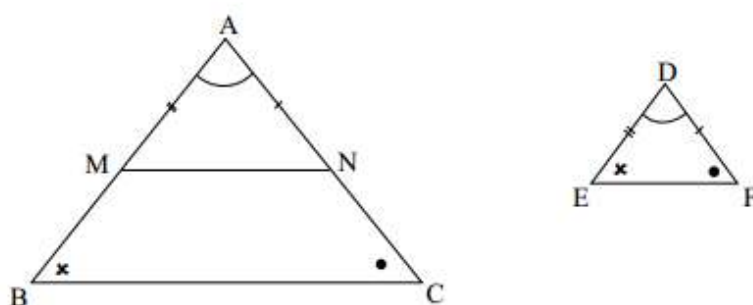


9.2.1	$NP = PL = 16$ [diag of $\parallel m$ / <i>hoeklyne van $\parallel m$</i>] $PT = 4$ $NP : PT = 16 : 4$ $= 4 : 1$	\checkmark S \checkmark R \checkmark S \checkmark answer (4)
9.2.2	$NM : MS = 4 : 1$ $NP : PT = NM : MS$ $KM \parallel RS$ [line divides two sides of Δ in prop / <i>Lyn verdeel 2 sye v Δ eweredig</i>] OR/OF [converse prop theorem / <i>omgekeerde lyn \parallel een sy v Δ</i>]	\checkmark S \checkmark R (2)
9.2.3	$\frac{RL}{KL} = \frac{TL}{LP}$ [prop theorem; $KM \parallel RS$ OR line \parallel one side of Δ / <i>Lyn \parallel een sy v Δ</i>] $RL = \frac{12 \times 21}{16}$ $= 15,75$	\checkmark S \checkmark R \checkmark S \checkmark answer (4)

	<p>OR / OF</p> <p>NM : MS = 4 : 1</p> <p>KR = MS = 5,25 [opp side of \parallel^m / teenoorst. sye van \parallel^m]</p> <p>KL = NM = 21</p> <p>RL + 5,25 = 21</p> <p>RL = 15,75</p>	<p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ answer</p> <p>(4)</p>
[16]		

QUESTION/VRAAG 10

10.1



10.1	<p>Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN.</p> <p>Proof:</p> <p>In $\triangle AMN$ and $\triangle DEF$</p> <p>AM = DE [Constr / Konstruksie]</p> <p>AN = DF [Constr / Konstruksie]</p> <p>$\hat{A} = \hat{D}$ [Given / Gegee]</p> <p>$\therefore \triangle AMN \cong \triangle DEF$ [s, \angle, s]</p> <p>$\therefore \hat{AMN} = \hat{E} = \hat{B}$</p> <p>MN \parallel BC [corresp \angle's are equal/ ooreenk. \angle e gelyk]</p> <p>$\frac{AB}{AM} = \frac{AC}{AN}$ [line \parallel one side of \triangle OR/OF prop theorem; MN \parallel BC]</p> <p>/ Lyn \parallel een sy v \triangle</p> <p>$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM=DE and AN=DF]</p>	<p>✓ Constr</p> <p>✓ S ✓ R</p> <p>✓ S / R</p> <p>✓ S ✓ R</p> <p>(6)</p>
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10.2.2	$\hat{G}_1 = 90^\circ$ [line from centre to midpt of chord / midpt. \odot ; midpt. koord]	✓ R (1)
10.2.3	In $\triangle GCD$ and $\triangle CDB$ $\hat{G}_2 = \hat{B}\hat{C}\hat{D} = 90^\circ$ $\hat{C}_3 = \hat{D}_2$ [∠s opp equal sides / ∠e teenoor gelyke syg] $\hat{G}\hat{D}\hat{C} = \hat{B}_3$ [sum of ∠s in \triangle / ∠e van \triangle] $\therefore \triangle GCD \parallel \triangle CDB$ [∠, ∠, ∠] $\therefore \frac{CD}{DB} = \frac{CG}{CD}$ [∠s] $\therefore CD^2 = CG \cdot DB$	✓ identifying \triangle s ✓ S ✓ S / R ✓ S OR ✓ R ✓ S (5)
10.2.4	$\frac{BC}{DB} = \frac{FB}{BC}$ [$\triangle FCB \parallel \triangle CDB$] $\therefore BC^2 = DB \cdot FB$ $CD^2 + BC^2 = CG \cdot DB + DB \cdot FB$ $DB^2 = DB(CG + FB)$ $DB = CG + FB$	✓ S ✓ R ✓ S ✓ sum ✓ $DB^2 = CD^2 + BC^2$ (5)
		[25]



BIBLIOGRAFIE

1	Mei/Junie 2022 – 2024 Nasienriglyne
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